ADAPTIVE STOCHASTIC RESONANCE & TRANSPORT ARRAY ULTRA WEAK WIRELESS SIGNAL PROCESSING

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Abstract
Adaptive Stochastic Resonance Array (ASRA) along with Adaptive Transport Array (ATA) preamplifier modules are exploited for ambient temperature (non-cryogenic) receiver front end recovery of Ultra Weak narrowband or wideband wireless signals which are subject to co-channel interference. The ASRA method uses pass band external additive or injecting signals in a weighted array framework for boosting the ultra weak wireless signal and the ATA method relies on instantaneous noise power analysis on a weighted array transport paths. The ASRA method is especially applicable for compact wireless devices such as wireless handsets and sensors, where the facilities for conventional antennas or Cryogenic setups are not available, but the ATA method requires further investigation before actual implementation. The preamplifier methods are insensitive to the transmission protocol and modulation scheme and they do not require extra bandwidth for their operation. The standard signal processing concepts such as mutual information stochastic resonance quality measures, gradient ascent and stochastic annealing, Kalman filtering and stochastic calculus have been used for updating the array weights or the injecting signal parameters.

I- Introduction

This presentation focuses on Adaptive Stochastic Resonance Array (ASRA) and Adaptive Transport Array (ATA) preamplifiers for the detection of ultra weak signals. The ultra weak signals are defined as desired signals that have receiver power levels below the noise level (floor) and ultra weak signal preamplifiers are employed before the LNAs at the wireless receiver in order to boost the desired signal power to levels above the noise floor. Section 2 provides a brief introduction on different ultra weak Signal handling modules. In section 3, a simple mathematics of ultra weak signal condition is presented, within the framework of traditional modulation format. Section 4 is devoted to Adaptive Stochastic Resonance Array (ASRA) preamplifier and it covers the architecture, resonance quality measure, weight and injecting signal controlling parameter update methods and the array performance. Section 5 is allocated to Adaptive Transport Array (ATA) preamplifier and the section covers the architecture along with the array weight assignment and cycle operation. Concluding remarks are included in section 6.

Ultra weak or sub threshold wireless signal processing is a fairly new branch in electronic communication and it came into existence for the fundamental objective of increasing the capacity of wireless users. The signal processing relies heavily on nano technology and semi ballistic electronic transport mechanism.

In order to increase the capacity of wireless users, the co-channel Signal Source Separation technology was developed to allow for the co-existence of wireless signals (of the same or different transmission protocols) in the same radio frequency band without increasing the band width. Moreover, many of the radio bands have been opened up for unlicensed operation and there is also a desire to increase the transmission distance or relax the line of sight requirements for microwave and millimeter wave bands.

However, the increase in the number of wireless transmissions with the conventional transmission power levels leads to wireless r.f. (radio frequency) power overloads. In fact, the standards for adding Source Separation modules to the wireless sets are subject to resolving the side issue of the r.f. power overloads.

The overload is an environmental health threat which can only be compensated by reduction in wireless transmitter power. Unfortunately, for a given transmission distance, the power reduction in wireless transmitters leads to attenuated signals with levels below the noise floor at the receiver. By employing multiple micro-cells and relays in the transmission path, it is possible to lower the transmitter power of the end users and have normal reception. In this case, the relays can communicate with other relays by a combination of terrestrial cabling and airway retransmissions.

If the relays use airways for the retransmission of signals, the r.f. power overload would still persist because the same amount of r.f. power is contributed to the overload. The r.f. power overload can be partially compensated by cabling and in fact, cabling is encouraged because it provides a redundant backup for portions of wireless networks. However, for reliable systems, cabling is not sufficient due to the terrestrial pathways and accessibility problems and a fully wireless system becomes mandatory in order to retain the reliability. Therefore, we still have to resolve the problem of detecting the attenuated wireless signals.

The problem of signal detection is compounded by the fact that the compact wireless modules that are used for personal communication or sensors do not have the necessary antenna power gains that are available to cellular, relay stations or fixed wireless components. Therefore, considering all of the mentioned factors, the Ultra Weak Signal Processing is ultimately required for detecting the wireless signals at power levels below the noise floor.

The ultra weak signal processing is basically required for the following reasons:

1- To detect wireless signals that are attenuated at levels below the noise floor due to low transmission power, long transmission distance, obstructions in indoor environments, etc.

2- To increase the transmission distance for wireless networks that have been traditionally used for short distances without increasing the transmission power

3- To allow for the deployment of massive wireless sensors and their networks at ambient temperature conditions

4- To increase the capacity for wireless service by operating in a low r.f. power environment and by eliminating the r.f. power overload

5- To allow for the inclusion of signal source separation modules in the wireless sets

II- Ultra Weak Signal Handling Modules or Options for Compact Wireless Sets

1- Nano Antennas

Nanotube antennas can be used to couple the external electromagnetic energy to the receiver front end [1]. They are extremely critical for the implementation of wireless hand set adaptive antenna array module.

2- Compact Nano Cavity Pre-amplifier

Space charge wave propagation theory of Hahn and Ramo [2],[3] can be employed on nano-cavities in order to amplify the ultra weak signal from the antenna ports. Each antenna port is connected to a compact nano cavity. In the cavity, an electron drift space can be set up by a coupled controlling D.C. electric field. By matching the group velocity of the weak signal field to the drift electron velocity through proper nano-cavity design, the kinetic energy of the drift electrons can be transferred to the ultra weak r.f. signal, which leads to signal amplification. The amplified r.f. signal is collected in a Catcher cavity.

Theories and approaches are available for confining and guiding electromagnetic energy through narrow channels with sub-wavelength transverse cross sections [4],[5]. The near zero ε ENZ materials have interesting potentials in efficiently squeezing and transmitting energy through narrow sub-wavelength region and effectively providing super-coupling between ports and/or waveguides. The ENZ materials can be properly synthesized at the desired frequency by embedding suitable inclusions in a host medium.

Also, Metal-Dielectric-Metal (MDM) structures with a dielectric region thickness of ~100 nm supports a propagating mode with a nanoscale modal size at a wavelength range extending from zero-frequency (DC) to visible [6].

3- Direct Electronic Noise Reducing Preamplifier

Although at a preliminary research stage, it is possible to reduce the intrinsic noise of the primary conduction path of the receiver (after the antenna ports) by reducing the electron-electron and electron-phonon scattering. In [7], coupled quantum dots in **phonon cavities** have been used to detect phonon quantum size effects in the electron transport. The quantum phase of an electron is randomized by inelastic scattering events with other electrons or with lattice vibrations (phonons). By tuning the dot level splitting via gate voltages, piezo-electric or deformation potential scattering can be drastically reduced.

4- Stochastic Resonance Preamplifiers

The Stochastic Resonance (SR) pre amplifier can be used for the detection of ultra weak narrowband or wideband wireless signals at ambient (non-cryogenic) temperature conditions. The sub-threshold signals can be detected at the receiver front end by externally injecting random signals and / or manipulating the internal noise level of the receiver front end. The SR methods do not require any extra bandwidth or any modification on the transmission protocol and they have a high degree of design flexibility for recovering the sub threshold signal.

A SR quality measure between the desired signal(s) and the SR output such as mutual information or normalized cross correlation is estimated based on either the modulation format or the reference sequences or the retrieved signals from the Source Separation module. Based on the SR quality measure, the injecting signal probability distribution parameters or signal formats are adaptively modified in order to increase the SR quality measure. The plot of the SR quality measure with respect to the adjusting parameter of the additive signal (or signals) resembles a resonance curve and the peak signifies the optimum operation point or resonance quality measure.[13, 21-31]

5- Adaptive Transport Array Preamplifier

This preamplifier is a recent addition to the ultra weak signal pre amplifier technology. At this primitive stage, it can not be practically implemented in the compact wireless set and it requires further research for optimizing and reducing the number of arrays.

Basically, the primary conduction path after the antenna / cavity port is divided into several weighted paths and the weighted paths are added to form an array. The instantaneous noise power of each path is estimated with respect to the desired signal power by using the output of the array which is accessible. During each update period, those paths that have instantaneous noise levels below the signal level will be heavily weighted and the other paths will not be emphasized. Effectively, the desired signal is captured at all times.

6- Signal Source Separation Modules

The Signal Source Separation (SS) modules [8-12] basically separate the desired signal from interfering signals by novel methods such as blind source separation which is related to the Independent Component Analysis and mutual information minimization methods.

The SS technology allows different interfering users to use the same or different transmission protocols in the relevant common frequency band **with out increasing the bandwidth**. The SS technology can distinguish signals that have different modulation format or signals that have the same modulation format but with different transmission protocol. But for signals that have the same transmission protocol and are not subject to a cellular subscriber assignment, either intrinsic identifying signal coding / transformation or Reference / Wireless Network Identification sequences should be available in order for the Source Separation module to identify the desired signal.

In spite of the advancements in SS modules and the increase in wireless capacity, the standards for adding SS modules to the wireless sets are subject to resolving the side issues such as the environmentally important wireless r.f. power overloads.

III- Ultra Weak Signal Condition

In this presentation, we will use the traditional 2 dimensional constellation type where the possible modulation signal set is specified by a discrete points, states or centroids s_K , k = 1 to Z in a normalized constellation space. The concepts in this presentation can be extended to advanced modulation formats as long as proper distance metrics are defined for the modulation points or states.

$S_R(t)=\mathrm{Re}\left\{\left[S_{RI}(t)+jS_{RQ}(t)\right]e^{jw_c t}\right\}$, where $S_R(t)$ is the received signal from an antenna port (3.1)

$S_R(t)=S_{RI}(t)\cos(w_c t)-S_{RQ}(t)\sin(w_c t)$, $S_{RI,Q}(t)=f_{MIX\,I,Q}\left(S_{DI,Q}(t),S_{IN\,I,Q}(t)\right)$, $N_{I,Q}(t)=\sigma_{I,Q}(t)Z_{I,Q}(t)$, where $S_{RI,Q}(t)$, $S_{DI,Q}(t)$, $S_{INI,Q}(t)$, $N_{I,Q}(t)$ are respectively, the base band versions of the received, desired, interference and the noise signals. I and Q are the typical inphase and quadrature components of the signals. The term $\sigma_{I,Q}(t)$ is the diffusion term and $Z(t)$ is the white noise random signal with unit variance. The term $f_{MIX}\left(S_{DI,Q}(t), S_{INI,Q}(t)\right)$ denotes the mixing operation between the co-channel desired and interfering signals.

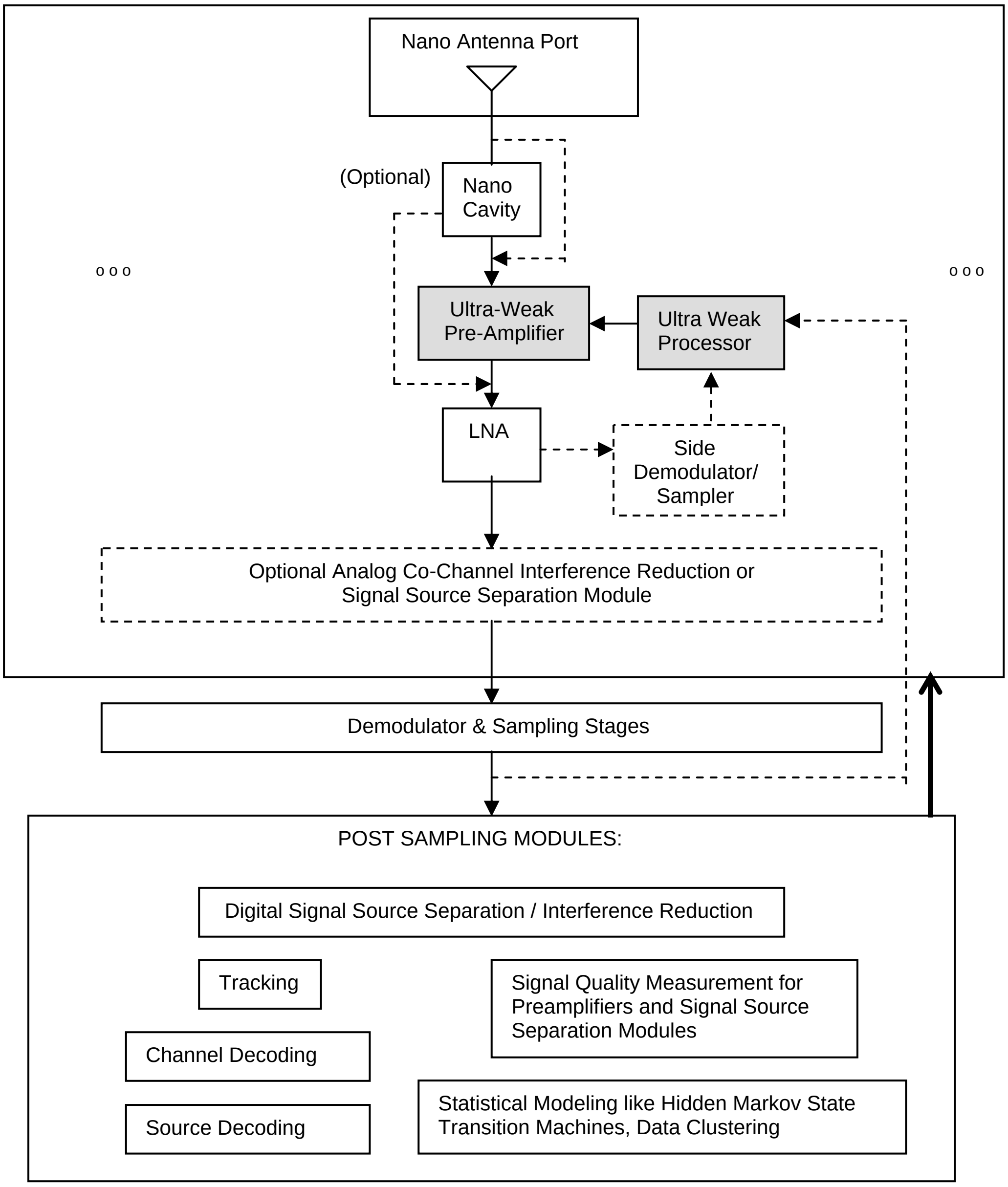

Fig. 1- Simplified Receiver Front End

In the simplest time invariant channel model, the baseband received signal can be represented by the following expression,

$$f_{MIX\,I,Q}\left(S_{DI,Q}(t), S_{INI,Q}(t)\right)$$

$$= h_{SD}(t)\, s_{DI,Q}(t) + \sum_{K=1}^{Lc-1} h_K(t)\, s_{DI,Q}(t-kT) + h_{IN}(t)\, s_{INI,Q}(t) \qquad (3.2)$$

where the $s(t)$ terms have unit powers and the coefficients $h(t)$ vary slowly with respect to the associated $s(t)$. The first and second terms represent the desired signal and the delayed ISI signals, respectively. The term $h_{INB}(t)\, s_{INI,Q}(t)$ represents the sum of the co channel interfered signals. The recovery of the desired $S_{DI,Q}(t)$ signal(s) by analog and post sampling signal source separation or co-channel interference reduction modules is denoted by the operator $\hat{f}_{SS}(\ )$. However, the signal recovery can be performed by conventional means only if the desired signal $h_{SD}(t)\, s_{DI,Q}(t)$ power is equal or above the noise floor.

The Ultra Weak Wireless Signal is defined by the condition

$$\overline{\left[h_{SD}(t)\, s_{DI,Q}(t)\right]^2} < \overline{N_{I,Q}^2(t)} \tag{3.3}$$

which implies that the signal power $\overline{\left[h_{SD}(t)\, s_{DI,Q}(t)\right]^2}$ is below the noise floor. Note that the received signal $f_{MIX}(t)$ may have average power levels above the noise floor, but the desired signal average power could still be below the noise floor. The power ratio for the desired and total received power is denoted by $\alpha_{D/MIX}^2 = \dfrac{\overline{\left[h_{SD}(t)\right]^2}}{\overline{\left[f_{MIX}(t)\right]^2}}$

By employing Ultra Weak wireless signal processing via ultra weak preamplifiers or other means, the desired signal average power would become greater than the noise floor and the source separation modules would be able to operate.

By labelling the Ultra Weak signal processing (or pre amplification) by an operator $\hat{f}_{UW}(\ )$, where

$(\hat{f}_{UW}\,[\, f_{MIX}(S_{DI,Q}(t), S_{IN\,I,Q}(t))\,,\, N_{I,Q}(t)])$ would have desired signal power above the noise floor, the recovered signal which is generated by the source separation module becomes:

$$\hat{s}_{DI,Q}(n) = \hat{f}_{SS}\,(\hat{f}_{UW}\,[\, f_{MIX}(S_{DI,Q}(t), S_{IN\,I,Q}(t))\,,\, N_{I,Q}(t)])\,,$$

The recovered signal is either mapped to a modulation constellation point, state or centroid s_K, k = 1 to Z or it is the constellation point itself, depending on the source separation method. The received signal data points are usually corrected by using the information from channel decoding and other modules.

IV- Adaptive Stochastic Resonance Array (ASRA) Preamplifier

A. ASRA Architecture

The ASRA method focuses on the pass band stochastic resonance external additive or injecting signal option for boosting the ultra weak or sub threshold signal. The ASRA method (Fig. 2) is the simplified extension of the combined Adaptive Stochastic Resonance [13], Supra-threshold Stochastic Resonance [16,17] and Adaptive Array [14,15] methods for processing modulated Ultra Weak wireless signals at the receiver.

As shown in Fig. 2, an array is formed by dividing the received signal from the antenna or the nano-cavity port into M transport paths. In each transport path (or array element) i, an additive random signal $S_i(t)$ with certain probability distribution is added to the received signal and the path is emphasised with weight multiplier $w_i(t)$. The multiplication is performed by a semi-ballistic nanostructure Gilbert cell such as nano FET Gilbert cell transistors with the minimal amount of field (voltage) to transport (current) translations [18,19]. Due to the ultra weak signal condition, the in phase and quadrature components are not usually implemented at the multiplication stage. Therefore, the weights $w_i(t)$ are usually not separated into I (in phase) and Q (quadrature) components.

The additive signal $S_i(t, \vec{\vartheta}_i, \vec{\xi}_i)$ is a random signal which is modulated in the same format as the desired signal and it possesses a probability distribution $p(S_i(t); \vec{\vartheta}_i)$ with controlling distribution parameters $\vec{\vartheta}_i$ and controlling modulation parameters $\vec{\xi}_i$ for the additive signal modulated format. The additive signal tries to imitate the desired signal $s_D(t)$ or at least tries to produce a signal which is aligned with the desired signal in order to boost the desired signal power to levels above the noise floor. Basically, the aim is to modify the additive signal probability parameters $\vec{\vartheta}_i$ such as the square root of the variance $\sigma(S_i(t))$ and to modify the additive signal modulation parameters $\vec{\xi}_i$ such as phase $\theta(S_i(t))$ in order to increase the correlation coefficient

$\rho(S_D(t), S_i(t)) = \frac{\langle s_D(t).S_i(t)\rangle}{\sqrt{\overline{s_D^2(t)}.\overline{S_i^2(t)}}}$ between the desired and additive signal or to equivalently decrease the average phase difference between the two signals.

It is not possible to track the desired signal adaptively by only using one additive signal at all times. Therefore, diversity is employed by using weighted M additive signals, $s_i(t)$ with i = 1 to M and by adjusting the controlling parameters of the M injecting signals $S_i(t)$ in order to maximize the correlation between the desired signal and the output of the array $y_T(t)$. In fact, it can be shown that the desired signal can be presented by a weighted mixture of random signals with different probability distribution or by signals that have the same p.d.f. format but with different parameterization, such as the weighted Gaussian distribution [20].

The maximum correlation between the desired signal and the array output is achieved indirectly by comparing the base band version of the array output $y_T(n)$ with the modulation states during tracking or the desired signal estimate $\hat{s}_D(n)$ during normal operation, calculating a SRQM (Stochastic Resonance Quality Measure), and by modifying the distribution parameters $\vec{\vartheta}_i$ and controlling modulation parameters $\vec{\xi}_i$ of $S_i(t, \vec{\vartheta}_i, \vec{\xi}_i)$, in order to maximize the SRQM. The plot of the SQRM verses the injecting signal modifying parameter such as the signal deviation resembles a resonance curve. The signal power of the array output will have power levels above the noise level. Therefore, the ASRA pre amplifier output can be processed by the Source Separation modules in order to estimate the desired signal.

The conduction path segment $S_R(t)$ from the output of each antenna or the nano-cavity port is divided into M equal size transport paths $S_{Ri}(t)$, i=1 to M having the following in-phase and quadrature base band format:

$$S_{Ri\,I}(t) = A_i(M)(S_{RI}(t)\cos\theta_i - S_{RQ}(t)\sin\theta_i), \tag{4.1}$$
$$S_{Ri\,Q}(t) = A_i(M)(S_{RI}(t)\sin\theta_i + S_{RQ}(t)\cos\theta_i) \text{ for i = 1 to M}$$

The terms $A_i(M)$ and θ_i are the attenuation and the phase shift angle for each transport path due to the division. The phase shift is usually neglected up to the microwave range, but it may be considered for the millimeter wave bands.

The mentioned terms can be determined from the geometry of the transport path division during actual implementation.

$$y_{T\,I,Q}(n) = \sum_{j=1}^{M} y_{j\,I,Q}(n) + v_{I,Q}(n), \tag{4.2}$$

where $y_{T\,I,Q}(n)$ is the sampled in phase & quadrature equivalent base band presentation for the array output, $v_{I,Q}(n)$ is the measurement error for the array output and

$$y_{j\,I,Q}(n) = g_{i\,I,Q}\left(w_i(n),[A_i(M).S_{R\,I,Q}(n) + S_{i\,I,Q}(n) + N_{i\,I,Q}(n)]\right) \tag{4.3}$$

is the sampled weighted base band format for the array components, i = 1 to M and $N_{i\,I,Q}(n)$ is the equivalent baseband presentation of the transport path noise $N_i(t)$ before the multiplication.

$$S_{R\,I,Q}(n) = h_{SD}(n)\,s_{DI,Q}(n) + \sum_{K=1}^{Lc-1} h_K(n)\,s_{D\,I,Q}(n-k) + h_{IN}(n)\,s_{IN\,I,Q}(n) \tag{4.4}$$

is the equivalent base band presentation for the received signal, and the function $g_{i\,I,Q}(\ )$ is used to signify the fact that the multiplication does not preserve the signal linear format because there may be desired signal or interfering signals components that have power levels below the noise floor and would not be amplified by the weights.

For each path, the injecting signal $S_i(t)$ can only be a partial duplicate of the desired signal $s_D(t)$ and therefore, portions of the desired signal would still have power levels below the noise floor. Therefore, by using the term $x_{i\,I,Q}(n)$ for the equivalent base band presentation of the multiplier input, the multiplication function $g_{i\,I,Q}(\ )$ satisfies the following condition,

$$x_{i\,I,Q}(n) < g_{i\,I,Q}\left(w_i(n), x_{i\,I,Q}(n)\right) \leq w_i(n).x_{i\,I,Q}(n) \tag{4.5}$$

Aside from the multiplication reduction, it is difficult to analyze the system dynamics directly due to the complexity of the internal noise mechanics and the difficulties in estimating the misalignment of interfering signals, injecting signal, and the internal noise signal with the ultra weak desired signal for each array element.

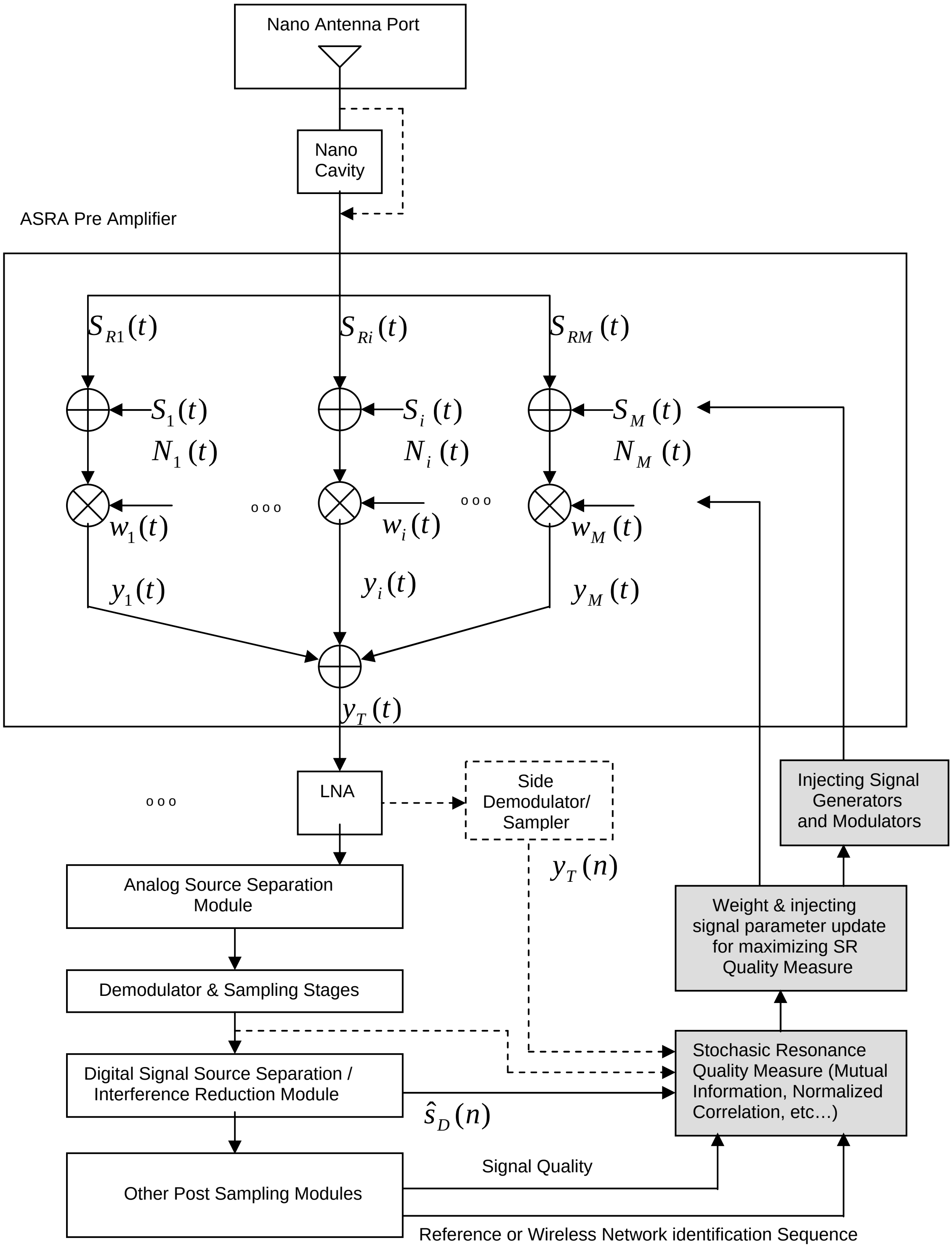

Fig. 2- Adaptive Stochastic Resonance Array

Therefore, we will resort to adaptive methods and the data samples that are available during tracking and acquisition in order to estimate the Stochastic Resonance Quality Measure (SRQM) and to modify the distribution parameters $\vec{\vartheta}_i$

and controlling modulation parameters $\vec{\xi}_i$ of $S_i(t, \vec{\vartheta}_i, \vec{\xi}_i)$ .The SRQM correlates the base band version of the array output $y_{T\,I,Q}(n)$ and the desired signal. The base band version of the array output $y_{T\,I,Q}(n)$ is always available, but the desired signal estimates $\hat{s}_D\,(n)$ become available by the Signal Source Separation module $\hat{s}_{D\,I,Q}(n) = \hat{f}_{SS}\left(y_{T\,I,Q}\,(n)\right)$ only after an initial successful operation of the ASRA preamplifier. Therefore, during the signal tracking mode, the modulation signal set (constellation points) itself will be used to optimize the parameters of ASRA.

B. ASRA Quality Measures

The Stochastic Resonance Quality Measure (SRQM) which is basically a measure of the correlation between the array output and the desired signal is maximized by adjusting the pdf parameters of the injecting signals such as the variance and by adjusting the injecting signal modulation phase. In the adaptive scheme, the quality measure is estimated during each operation cycle in order to examine its gradient. The SRQM [21-23] can be estimated by many methods but the focus in this presentation will be on Standard Mutual Information and Quadratic Mutual Information method. The Normalized Correlation measure and other methods will not be covered. By estimating the SRQM, we have virtually alleviated the requirement for examining the internal dynamics of the ASRA preamplifier.

1- Standard Mutual Information

Standard Mutual Information can be used as a measure of Stochastic Resonance without the requirement of desired signal estimates $\hat{s}_{D\,I,Q}(n)$ and it is usually employed at the tracking phase, where the Source Separation module is not operating. This SRQM which is dependent on the modulation format and the constellation assignment will activate the ASRA so that the Source Separation Module can extract the desired signals.

The source separation technology allows different interfering users to use the same or different transmission protocols in the relevant common frequency band. If there is an cooperative channel agreement to use the same modulation or transmission protocol, then either intrinsic identifying signal coding / transformation or Reference / Wireless Network Identification sequences should be available in order for the Source Separation module to identify the desired signal. If intrinsic signal coding is used to identify the desired signal, the SRQM operates by using the modulation format and the Source Separation Module will extract the desired signal. If reference or identification sequences are used to distinguish between users, the ASRA and Source Separation modules will try to lock to the sequences at the tracking phase.

The standard Mutual Information measure requires only the knowledge of the sampled base band version of the array output $y_{T\,I,Q}(n)$ and the modulation format in the form of signal constellation or centroids or states. The sampled $y_{T\,I,Q}(n)$ are assumed to be properly scaled to match the signal constellation format. Let Y be the set for the array output with elements $y_{T\,I,Q}(n)$, and let S be the set for the desired signal possible points on the modulation constellation or states or centroids with elements $s_K = (s_{K\,I}, s_{K\,Q})$, k = 1 to Z. The probability distribution for Y, p(y) can be represented in either discrete or continuous format.

For continuous presentation of Y, the standard mutual information is

$$I_{STD}(S,Y) = \int_Y \sum_{k=1}^{Z} p(s_k, y) \log \frac{p(s_k, y)}{p(s_k)\,p(y)} dy \qquad (4.6)$$
$$= \int_Y \sum_{k=1}^{Z} p(y \mid s_k)\,p(s_k) \log \frac{p(y \mid s_k)}{p(y)} dy$$

For discrete presentation of Y with N+1 samples, the standard mutual information becomes

$$I_{STD}(S,Y) = \sum_{m=0}^{N} \sum_{k=1}^{Z} p(s_k, y) \log \frac{p(s_k, y(n-m))}{p(s_k)\,p(y(n-m))} \qquad (4.7)$$

$$= -\sum_{m=0}^{N} p(y_{T\,I,Q}(n-m)) \log p(y_{T\,I,Q}(n-m)) + \sum_{k=1}^{Z} p(s_k) \sum_{m=0}^{N} p(y_{T\,I,Q}(n-m) \mid s_k) \log p(y_{T\,I,Q}(n-m) \mid s_k)$$

In general, $p(s_k) = 1/Z$ and by using the Parzen Window method for N+1 samples of Y, the probability distribution function for Y which will be available in time period n will be

$$p(y_{T\,I,Q}) = \frac{1}{N} \sum_{m=0}^{N} G(y_{T\,I,Q} - y_{T\,I,Q}(n-m), \sigma_y^2),$$

where the Gaussian Kernel is defined as

$$G(y_{T\,I,Q} - y_{T\,I,Q}(n-m), \sigma_y^2) = \frac{1}{2\pi\sigma_y} \exp\left\{ \frac{(y_I - y_{T\,I}(n-m))^2 + (y_Q - y_{Q\,I}(n-m))^2}{2\sigma_y^2} \right\} \quad (4.8)$$

and the conditional densities are

$$p(y_{T\,I,Q}(n-m) \mid s_k) = \frac{1}{2\pi\sigma_s} \exp\left\{ \frac{(s_{K\,I} - y_{T\,I}(n-m))^2 + (s_{K\,Q} - y_{Q\,I}(n-m))^2}{2\sigma_s^2} \right\} \quad (4.9)$$

Note that we have not used any reference data or the estimated desired signal $\hat{s}_{D\,I,Q}(n)$ from the Source Separation module. The Quadratic Mutual Information and Normalized Correlation methods that are covered in this presentation will use the estimated desired signals $\hat{s}_{D\,I,Q}(n)$ along with the ASRA outputs $y_{T\,I,Q}(n)$.

2- Quadratic Mutual Information

Inspired by Renyi's quadratic mutual information measure, researchers have come up with other information theoretic distance measures to estimate the mutual information. Let Y be the set for the array output with elements $y_{T\,I,Q}(n)$, and let S be the set for the estimated desired signals $\hat{s}_{D\,I,Q}(n)$ generated by the source separation modules. The Cauchy Schwartz quadratic Mutual Information [23] (CS-QMI) measure which is based on the Cauchy-Schwartz inequality is

$$I_{CS}(S,Y) = \log \frac{\int_Y \int_S p^2(s,y)dsdy . \int_Y \int_S p^2(s) p^2(y) dsdy}{\left[\int_Y \int_S p(s,y) p(s) p(y) dsdy \right]^2} \quad (4.10)$$

The mutual information is simplified by using the Parzen window method with the Gaussian Kernel for the joint and marginal distributions.

$$p(s,y) = \frac{1}{N} \sum_{m=0}^{N} G(s_{I,Q} - \hat{s}_{D\,I,Q}(n-m), \sigma_s^2) G(y_{I,Q} - y_{T\,I,Q}(n-m), \sigma_y^2) \quad (4.11)$$

$$p(s) = \frac{1}{N} \sum_{m=0}^{N} G(s_{I,Q} - \hat{s}_{D\,I,Q}(n-m), \sigma_s^2) \text{ and } p(y) = \frac{1}{N} \sum_{m=0}^{N} G(y_{I,Q} - y_{T\,I,Q}(n-m), \sigma_y^2)$$

The CS-QMI measure is reduced to the following expression,

$$I_{CS}(S,Y) = \log \frac{v(s,y) v_1(s) v_2(y)}{v_{nc}(s,y)}$$

where the variables $v(s,y)$, $v_1(s)$, $v_2(y)$ and $v_{nc}(s,y)$ are defined below:

$$v(s,y) = \frac{1}{N^2} \sum_{i=0}^{N} \sum_{j=0}^{N} [G(\hat{s}_{D\,I,Q}(n-i) - \hat{s}_{D\,I,Q}(n-j), 2\sigma_s^2) . G(y_{T\,I,Q}(n-i) - y_{T\,I,Q}(n-j), 2\sigma_y^2)] \quad (4.12)$$

$$v_1(s) = \frac{1}{N^2} \sum_{i=0}^{N} \sum_{j=0}^{N} G(\hat{s}_{D\,I,Q}(n-i) - \hat{s}_{D\,I,Q}(n-j), 2\sigma_s^2)$$

$$v_2(y) = \frac{1}{N^2} \sum_{i=0}^{N} \sum_{j=0}^{N} G(y_{T\,I,Q}(n-i) - y_{T\,I,Q}(n-j), 2\sigma_y^2)$$

$$v_{nc}(s,y) = \frac{1}{N} \sum_{i=0}^{N} \left\{ \left[\frac{1}{N} \sum_{j=0}^{N} G(\hat{s}_{D\,I,Q}(n-i) - \hat{s}_{D\,I,Q}(n-j), 2\sigma_s^2) \right] . \left[\frac{1}{N} \sum_{j=0}^{N} G(y_{T\,I,Q}(n-i) - y_{T\,I,Q}(n-j), 2\sigma_y^2) \right] \right\}$$

C. ASRA Weight and Injecting Signal Controlling Parameter Update

1- Adaptive Controlling Parameter Estimation

By using the estimates of the Stochastic Resonance Quality Measure (SRQM) and other information [24], it is possible to update the weights $w_i(n)$ of the array, the controlling distribution parameters $\vec{\vartheta}_i$ and the parameters $\vec{\xi}_i$ of the additive random signals $S_i(t, \vec{\vartheta}_i, \vec{\xi}_i)$ with probability distributions $p(S_i(t); \vec{\vartheta}_i)$. The general terminology $I_{S,Y}(n)$ will be used instead of $I_{STD}(S,Y)$ and $I_{CS}(S,Y)$ to indicate the measured SRQM between the array output $y_{T\,I,Q}(n)$ and the desired signal $S_{D\,I,Q}(t)$ at the update period n. The usual probability distribution controlling parameter is the square root of variance or deviation $\sigma(S_i(t))$.

The gradient of the quality measure with respect to the array weights is approximated by the following expression:

$$\begin{aligned}\frac{\Delta I_{S,Y}(n)}{\Delta w_i(n)} &= \frac{I_{S,Y}(n-1)-I_{S,Y}(n-2)}{y_{T\,I}(n-1)-y_{T\,I}(n-2)}.\frac{\Delta y_{T\,I}}{\Delta y_{i\,I}}.\frac{\hat{y}_{i\,I}(n-1)-\hat{y}_{i\,I}(n-2)}{w_i(n-1)-w_i(n-2)} \\ &+\frac{I_{S,Y}(n-1)-I_{S,Y}(n-2)}{y_{T\,Q}(n-1)-y_{T\,Q}(n-2)}.\frac{\Delta y_{T\,Q}}{\Delta y_{i\,Q}}.\frac{\hat{y}_{i\,Q}(n-1)-\hat{y}_{i\,Q}(n-2)}{w_i(n-1)-w_i(n-2)}\end{aligned} \tag{4.13}$$

where $\hat{y}_{i\,I}(k)$ and $\hat{y}_{i\,Q}(k)$ are the in phase and quadrature a posteriori estimates of the weighted array components. We assumed that the gradient with respect to the in phase and quadrature components of the weighted paths will move in the same direction due to the fact that desired signal in phase and quadrature components experience the same channel conditions. However, if that is not the case, then the methods for multiple optimizations [32-33] should be employed. The terms $\frac{\Delta y_{T\,I,Q}}{\Delta y_{i\,I,Q}}$ can be estimated from $y_{T\,I,Q}(n) = \sum_{j=1}^{M} y_{j\,I,Q}(n) + v_{I,Q}(n)$, as follows:

$$v_{I,Q}(n) \approx y_{T\,I,Q}(n) - \sum_{j=1}^{M} \hat{y}_{j\,I,Q}(n)$$

$$\begin{aligned}\frac{\Delta y_{T\,I,Q}}{\Delta y_{i\,I,Q}} &\approx \frac{\partial}{\partial y_{i\,I,Q}}\left[\sum_{j=1}^{M} y_{j\,I,Q}(n) + v_{I,Q}(n)\right] \\ &\approx 1+\frac{y_{T\,I,Q}(n-1)-\sum_{j=1}^{M}\hat{y}_{j\,I,Q}(n-1)-y_{T\,I,Q}(n-2)+\sum_{j=1}^{M}\hat{y}_{j\,I,Q}(n-2)}{\hat{y}_{i\,I,Q}(n-1)-\hat{y}_{i\,I,Q}(n-2)}\end{aligned} \tag{4.14}$$

The gradient of the quality measure with respect to the injecting signals $S_i(t)$ probability distribution controlling parameter $\vartheta_{i\,I,Q}$ belonging to $\vec{\vartheta}_i$ is approximated by the following expression:

$$\frac{\Delta I_{S,Y}(n)}{\Delta \vartheta_{i\,I,Q}} = \frac{I_{S,Y}(n-1)-I_{S,Y}(n-2)}{y_{T\,I,Q}(n-1)-y_{T\,I,Q}(n-2)}.\frac{\Delta y_{T\,I,Q}}{\Delta y_{i\,I,Q}}.\frac{\hat{y}_{i\,I,Q}(n-1)-\hat{y}_{i\,I,Q}(n-2)}{\vartheta_{i\,I,Q}(n-1)-\vartheta_{i\,I,Q}(n-2)} \tag{4.15}$$

The simplest form for updating the weights $w_i(n)$ and the injecting signal probability distribution controlling parameters $\vartheta_{i\,I,Q}$ such as the deviation $\sigma(S_i(t))$ belonging to $\vec{\vartheta}_i$ would be the following steepest ascent expressions:

$$\begin{aligned}w_i(n) &= w_i(n-1) + \eta(w_i(n))\frac{\Delta I_{S,Y}(n)}{\Delta w_i(n)} \\ &+\sigma_{Ann}(w_i(n)).\Delta B(w_i(n)) + 0.5\sigma_{Ann}(w_i(n))\sigma'_{Ann}(w_i(n))\left(\Delta B^2(w_i(n)) - \Delta T\right) \\ \vartheta_{i\,I,Q}(n) &= \vartheta_{i\,I,Q}(n-1) + \eta(\vartheta_{i\,I,Q}(n))\frac{\Delta I_{S,Y}(n)}{\Delta \vartheta_{i\,I,Q}} \\ &+\sigma_{Ann}(\vartheta_{i\,I,Q}(n)).\Delta B_{I,Q}(\vartheta_{i\,I,Q}(n)) + 0.5\sigma_{Ann}(\vartheta_{i\,I,Q}(n))\sigma'_{Ann}(\vartheta_{i\,I,Q}(n))\left(\Delta B^2_{I,Q}(\vartheta_{i\,I,Q}(n)) - \Delta T\right)\end{aligned} \tag{4.16}$$

where $\eta(..(n))$ are the gradient adaptation factors, $\sigma_{Ann}(...(n))$ are the stochastic annealing coefficients, the Brownian increments $\Delta B(..(n)) = B(..(n)) - B(..(n-1))$, $B(..(n))$ are Brownian motion white Gaussian noise generators with unit variance, and $\sigma'_{Ann}(..(n))$ are the annealing coefficient discrete derivative with respect to time increment ΔT between samples. The annealing terms [34-36] were included to escape from local maximum points.

The gradient of the quality measure with respect to the injecting signals $S_i(t)$ controlling modulation parameters $\vec{\xi}_i$ such as the modulation phase is analyzed after the procedure for estimating the pre weighted array components.

2- Base band equivalent signal estimates for the weighted and pre-weighted array components

In order to use the mentioned simple updates or to use more advanced formats, the estimates for the weighted array components $\hat{y}_{i\,I,Q}(n-k)$, k = 1, 2, … are required. Moreover, better estimates for the gradients and other updating procedures can be used if the pre weighted array path components $x_{i\,I,Q}(n-k)$ can also be estimated. For this purpose, we use the relationship between $y_{i\,I,Q}(n)$ and the available injecting signal sampled base band format $S_{i\,I,Q}(n)$ along with the sampled base band version of the array output $y_{T\,I,Q}(n)$.

As mentioned before, for each transport path, the desired signal or interfering signal components still have power levels below the noise floor even after the addition of the injecting signal. Therefore, only a portion of the pre weighted signal $x_{i\,I,Q}(n)$ would be amplified by the weight $w_i(t)$ and the rest would not be amplified. If $\alpha_{i\,I,Q}(n)$ denotes the fraction of $x_{i\,I,Q}(n)$ that is amplified by the weight at the operation cycle n and by assigning $B_{i\,I,Q}(n) = A_i(M).S_{R\,I,Q}(n) + N_{i\,I,Q}(n)$, the weighted array element becomes

$$\begin{aligned} y_{i\,I,Q}(n) &= g_{i\,I,Q}\big(w_i(n), [A_i(M).S_{R\,I,Q}(n) + S_{i\,I,Q}(n) + N_{i\,I,Q}(n)]\big) \\ &= g_{i\,I,Q}\big(w_i(n), x_{i\,I,Q}(n)\big) \\ &= \alpha_{i\,I,Q}(n) w_i(n)\big(B_{i\,I,Q}(n) + S_{i\,I,Q}(n)\big) + \big(1-\alpha_{i\,I,Q}(n)\big)\big(B_{i\,I,Q}(n) + S_{i\,I,Q}(n)\big) \end{aligned} \tag{4.17}$$

By defining further variables,

$$D_{i\,I,Q}(n) = \alpha_{i\,I,Q}(n) B_{i\,I,Q}(n) \text{ and } C_{i\,I,Q}(n) = \big(1-\alpha_{i\,I,Q}(n)\big) B_{i\,I,Q}(n), \tag{4.18}$$

$$y_{j\,I,Q}(n) = \Big(1 + S_{i\,I,Q}(n)\big(D_{i\,I,Q}(n) + C_{i\,I,Q}(n)\big)^{-1}\Big)\big(w_i(n) D_{i\,I,Q}(n) + C_{i\,I,Q}(n)\big)$$

and by using the available array output $y_{T\,I,Q}(n) = \sum_{j=1}^{M} y_{j\,I,Q}(n) + v_{I,Q}(n)$, the required weighted array estimates $\hat{y}_{iQ}(n-k)$, k = 1, 2, …can be determined indirectly by estimating the variables $D_{i\,I,Q}(n)$ and $C_{i\,I,Q}(n)$.

The extended Kalman Filtering method [37] can be used to estimate the variables $D_{i\,I,Q}(n)$ and $C_{i\,I,Q}(n)$. For simplicity, the I and the Q subscripts will be emitted.

Let $D(k) = [D_1(k)\, D_2(k) ... D_M(k)]$ and $C(k) = [C_1(k)\, C_2(k) ... C_M(k)]$ be the vector format for the mentioned variables. Also, let $\hat{D}(k) = [\hat{D}_1(k)\, \hat{D}_2(k) ... \hat{D}_M(k)]$ and $\hat{C}(k) = [\hat{C}_1(k)\, \hat{C}_2(k) ... \hat{C}_M(k)]$ be the vectors for the a posteriori estimates with the corresponding a priori estimates $\tilde{D}(k)$ and $\tilde{C}(k)$ and update matrices $\Lambda_{D1}, \Lambda_{D2}, \Lambda_{C1}$ and Λ_{C2}. The simplest a priori update mechanism is $\tilde{D}(n) = \Lambda_{D1}\hat{D}(n-1) + \Lambda_{D2}\hat{D}(n-2)$, $\tilde{C}(n) = \Lambda_{C1}\hat{C}(n-1) + \Lambda_{C2}\hat{C}(n-2)$, and Kalman Filtering can be used to calculate the a posteriori estimates $\hat{D}(k)$ and $\hat{C}(k)$.

The KFPE (Kalman Filter Preliminary Equations) are listed below:

$$\tilde{y}_T(n) = \sum_{j=1}^{M} \Big(1 + S_j(n)\big(\tilde{D}_j(n) + \tilde{C}_j(n)\big)^{-1}\Big)\big(w_j(n)\tilde{D}_j(n) + \tilde{C}_j(n)\big) \tag{4.19}$$

as the a priori estimate of the array output

$$D(n) \approx \tilde{D}(n) + \Lambda_{D1}(D(n) - \hat{D}(n-1)) + w_D(n-1), \quad (4.20)$$

where $w_D(n-1)$ as the process noise with covariance $Q_D(n-1)$,

$$C(n) \approx \tilde{C}(n) + \Lambda_{C1}(C(n) - \hat{C}(n-1)) + w_C(n-1), \quad (4.21)$$

where $w_C(n-1)$ as the process noise with covariance $Q_C(n-1)$,

$$y_T(n) \approx \tilde{y}_T(n) + H_D(D(n) - \tilde{D}(n)) + v_D(n), \quad (4.22)$$

where $v_D(n)$ as the measured noise due to $D(n)$ contribution with variance $R_D(n)$,

$$y_T(n) \approx \tilde{y}_T(n) + H_C(C(n) - \tilde{C}(n)) + v_C(n), \quad (4.23)$$

where $v_C(n)$ as the measured noise due to $C(n)$ contribution with variance $R_C(n)$,

$$H_D(n) = \left[\frac{\partial y_T(n)}{\partial D_1(n)}, \frac{\partial y_T(n)}{\partial D_2(n)}, \ldots, \frac{\partial y_T(n)}{\partial D_M(n)}\right] \text{ and } H_C(n) = \left[\frac{\partial y_T(n)}{\partial C_1(n)}, \frac{\partial_T(n)}{\partial C_2(n)}, \ldots, \frac{\partial y_T(n)}{\partial C_M(n)}\right] \quad (4.24)$$

are respectively the Jacobians of the array output with respect to $D(n)$ evaluated at $\tilde{D}(n)$, and $C(n)$ evaluated at $\tilde{C}(n)$.

$$H_{Di} = -S_i(n)\left(\tilde{D}_i(n) + \tilde{C}_i(n)\right)^{-2}\left(w_i(n)\tilde{D}_i(n) + \tilde{C}_i(n)\right) + w_i(n)\left(1 + S_i(n)\left(\tilde{D}_i(n) + \tilde{C}_i(n)\right)^{-1}\right) \quad (4.25)$$

$$H_{Ci} = -S_i(n)\left(\tilde{D}_i(n) + \tilde{C}_i(n)\right)^{-2}\left(w_i(n)\tilde{D}_i(n) + \tilde{C}_i(n)\right) + \left(1 + S_i(n)\left(\tilde{D}_i(n) + \tilde{C}_i(n)\right)^{-1}\right)$$

$\tilde{e}(D(n)) = D(n) - \tilde{D}(n)$ and $\tilde{e}(C(n)) = C(n) - \tilde{C}(n)$ are the a priori estimate errors (4.26)

$\tilde{P}_D(n) = E[\tilde{e}(D(n))\,\tilde{e}(D(n))^T]$ and $\tilde{P}_C(n) = E[\tilde{e}(C(n))\,\tilde{e}(C(n))^T]$ (4.27)

are the a priori estimate error covariances,

$\hat{e}(D(n)) = D(n) - \hat{D}(n)$ and $\hat{e}(C(n)) = C(n) - \hat{C}(n)$ are the a posteriori prediction errors (4.28)

$\hat{P}_D(n) = E[\hat{e}(D(n))\,\hat{e}(D(n))^T]$ and $\hat{P}_C(n) = E[\hat{e}(C(n))\,\hat{e}(C(n))^T]$ (4.29)

are the a posteriori estimate error covariances,

The a posteriori estimates are related to the a priori estimates by the Kalman gain,

$$\begin{aligned}\hat{D}(n) &= \tilde{D}(n) + K_D(n)(y_T(n) - \tilde{y}_T(n)) \quad (4.30)\\ &= \tilde{D}(n) + K_D(n)(H_D(D(n) - \tilde{D}(n)) + v_D(n))\end{aligned}$$

where $K_D(n)$ is the Kalman gain

$$\begin{aligned}\hat{C}(n) &= \tilde{C}(n) + K_C(n)(y_T(n) - \tilde{y}_T(n)) \quad (4.31)\\ &= \tilde{C}(n) + K_C(n)(H_C(C(n) - \tilde{C}(n)) + v_C(n))\end{aligned}$$

where $K_C(n)$ is the Kalman gain

The **KFUE (Kalman Filter Update Equations)** are stated below, (4.32)

$\tilde{P}_D(n) = \Lambda_{D1}\hat{P}_D(n-1)\Lambda_{D1}{}^T + Q_D(n-1)$ is the update for $D(n)$ a priori estimate error covariance,

$\tilde{P}_C(n) = \Lambda_{C1}\hat{P}_C(n-1)\Lambda_{C1}{}^T + Q_C(n-1)$ is the update for $C(n)$ a priori estimate error covariance,

$K_D(n) = \tilde{P}_D(n)H_D^T(n)[H_D(n)\tilde{P}_D(n)H_D^T(n) + R_D(n)]^{-1}$ is the filter gain for $D(n)$,

$K_C(n) = \tilde{P}_C(n)H_C^T(n)[H_C(n)\tilde{P}_C(n)H_C^T(n) + R_C(n)]^{-1}$ is the filter gain for $C(n)$,

$\hat{D}(n) = \tilde{D}(n) + K_D(n)(y_T(n) - \tilde{y}_T(n))$ is the a posteriori estimate for $D(n)$,

$\hat{C}(n)=\tilde{C}(n)+K_C(n)(y_T(n)-\tilde{y}_T(n))$ is the a posteriori estimate for $C(n)$,
$\hat{P}_D(n)=[1-K_D(n)H_D(n)]\tilde{P}_D(n)$ is the update for $D(n)$ a posteriori estimate error covariance,
$\hat{P}_C(n)=[1-K_C(n)H_C(n)]\tilde{P}_C(n)$ is the update for $C(n)$ a posteriori estimate error covariance,

In order to calculate the covariances $Q_D(n-1)$ and $Q_C(n-1),$ we can us the estimates

$$\hat{w}_D(n-k)=\hat{D}(n-k)-\tilde{D}(n-k)-\Lambda_{D1}(\hat{D}(n-k)-\hat{D}(n-k)),\ \text{k = 1, 2,…} \tag{4.33}$$

$$\hat{w}_C(n-k)=\hat{C}(n-k)-\hat{C}(n-k)-\Lambda_{C1}(\hat{C}(n-k)-\hat{C}(n-k)),\ \text{k = 1, 2,…}$$

In order to calculate the variances $R_D(n)$ and $R_C(n)$,we can us the estimates

$$\hat{v}_D(n-k)=y_T(n-k)-\tilde{y}_T(n-k)-H_D(\hat{D}(n-k)-\tilde{D}(n-k)),\ \text{k = 1, 2, …} \tag{4.34}$$

$$\hat{v}_C(n-k)=y_T(n-k)-\tilde{y}_T(n-k)-H_C(\hat{C}(n-k)-\tilde{C}(n-k)),\ \text{k = 1, 2, …}$$

The $\Lambda_{D1},\Lambda_{D2},\Lambda_{C1}$ and Λ_{C2} matrixes which are used for the a priori estimates of $D(n)$ and $C(n)$ are usually diagonal and are updated in order to minimize the mean square error between the a priori and a posteriori estimates of $D(n)$ and $C(n)$.

By using the a posteriori estimates $\hat{D}(k)$ and $\hat{C}(k)$, the weighted array estimates become

$$\hat{y}_{iI,Q}(n)=\left(1+S_{iI,Q}(n)\left(\hat{D}_{iI,Q}(n)+\hat{C}_{iI,Q}(n)\right)^{-1}\right).\left(w_i(n)\hat{D}_{iI,Q}(n)+\hat{C}_{iI,Q}(n)\right) \tag{4.35}$$

The pre weighted array estimates are given by

$$\hat{x}_{iI,Q}(n)=\hat{D}_{iI,Q}(n)+\hat{C}_{iI,Q}(n)+S_{iI,Q}(n) \tag{4.36}$$

The fraction of un-weighted array element signal which is amplified by the weights is given by

$$\hat{\alpha}_{iI,Q}(n)=\hat{D}_{iI,Q}(n).(\hat{D}_{iI,Q}(n)+\hat{C}_{iI,Q}(n))^{-1} \tag{4.37}$$

3- Improved Controlling Parameter Estimation (including additive signal modulation phase)

A better estimate for the gradient of the quality measure with respect to the injecting signals $S_i(t)$ controlling parameter $\vartheta_{iI,Q}$ belonging to $\vec{\vartheta}_i$ is expressed below,

$$\frac{\Delta I_{S,Y}(n)}{\Delta\vartheta_{iI,Q}}=\frac{I_{S,Y}(n-1)-I_{S,Y}(n-2)}{y_{TI,Q}(n-1)-y_{TI,Q}(n-2)}.\frac{\Delta y_{TI,Q}}{\Delta y_{iI,Q}}.\frac{\partial y_{iI,Q}(n)}{\partial x_{iI,Q}(n)}.\frac{\hat{x}_{iI,Q}(n-1)-\hat{x}_{iI,Q}(n-2)}{\vartheta_{iI,Q}(n-1)-\vartheta_{iI,Q}(n-2)} \tag{4.38}$$

$$\frac{\partial y_{iI,Q}(n)}{\partial x_{iI,Q}(n)}\approx\hat{\alpha}_{iI,Q}(n)w_i(n)+(1-\hat{\alpha}_{iI,Q}(n))\ \text{due to}\ y_{iI,Q}(n)=\alpha_{iI,Q}(n)w_i(n)x_{iI,Q}(n)+(1-\alpha_{iI,Q}(n))x_{iI,Q}(n)$$

Similarly, a better estimate for the gradient of the quality measure with respect to the array weight is expressed below,

$$\begin{aligned}\frac{\Delta I_{S,Y}(n)}{\Delta w_i(n)}=&\frac{I_{S,Y}(n-1)-I_{S,Y}(n-2)}{y_{TI}(n-1)-y_{TI}(n-2)}.\frac{\Delta y_{TI}}{\Delta y_{iI}}.\frac{\partial y_{iI}(n)}{\partial x_{iI}(n)}.\frac{\hat{x}_{iI}(n-1)-\hat{x}_{iI}(n-2)}{w_i(n-1)-w_i(n-2)}\\&+\frac{I_{S,Y}(n-1)-I_{S,Y}(n-2)}{y_{TQ}(n-1)-y_{TQ}(n-2)}.\frac{\Delta y_{TQ}}{\Delta y_{iQ}}.\frac{\partial y_{iQ}(n)}{\partial x_{iQ}(n)}.\frac{\hat{x}_{iQ}(n-1)-\hat{x}_{iQ}(n-2)}{w_i(n-1)-w_i(n-2)}\end{aligned} \tag{4.39}$$

where the assumption regarding the Pareto optimality between the gradient with respect to the in phase and the quadrature components still holds.

For the modulated format of the additive signal $S_i(t,\vec{\vartheta}_i,\vec{\xi}_i)$,the gradient of the quality measure with respect to the injecting signal controlling modulation parameter such as the modulation phase $\theta(S_i,n)$ belonging to the vector $\vec{\xi}_i$ is approximated by the following expression:

$$\frac{\Delta I_{S,Y}(n)}{\Delta\theta(S_i,n)}=\frac{I_{S,Y}(n-1)-I_{S,Y}(n-2)}{y_{T\,I}(n-1)-y_{T\,I}(n-2)}.\frac{\Delta y_{T\,I}}{\Delta y_{i\,I}}.\frac{\partial y_{i\,I}(n)}{\partial x_{i\,I}(n)}.\frac{\hat{x}_{i\,I}(n-1)-\hat{x}_{i\,I}(n-2)}{\theta(S_i,n-1)-\theta(S_i,n-2)}$$
$$+\frac{I_{S,Y}(n-1)-I_{S,Y}(n-2)}{y_{T\,Q}(n-1)-y_{T\,Q}(n-2)}.\frac{\Delta y_{T\,Q}}{\Delta y_{i\,Q}}.\frac{\partial y_{i\,Q}(n)}{\partial x_{i\,Q}(n)}.\frac{\hat{x}_{i\,Q}(n-1)-\hat{x}_{i\,Q}(n-2)}{\theta(S_i,n-1)-\theta(S_i,n-2)} \quad (4.40)$$

where the controlling modulation phase $\theta(S_i\,,n)$ is used for the additive signal modulation format such as

$$S_i(t)=S_{i\,I}(t)\cos(w_c t+\theta(S_i))-S_{i\,Q}(t)\sin(w_c t+\theta(S_i)) \quad (4.41)$$
$$=[S_{i\,I}^2(t)+S_{iQ}^2(t)]^{1/2}\cos[w_c t+\theta(S_i)+\arctan\frac{S_{iQ}(t)}{S_{i\,I}(t)}]$$

$$\theta(S_i,n)=\theta(S_i,n-1)+\eta(\theta(S_i,n))\frac{\Delta I_{S,Y}(n)}{\Delta\theta(S_i,n)} \quad (4.42)$$
$$+\sigma_{Ann}(\theta(S_i,n)).\Delta B(\theta(S_i,n))+0.5\sigma_{Ann}(\theta(S_i,n))\sigma'_{Ann}(\theta(S_i,n))\left(\Delta B^2(\theta(S_i,n))-\Delta T\right)$$

where $\sigma_{Ann}(\theta(S_i\,,n))$ is the stochastic annealing coefficient, the Brownian increment [42]. $\Delta B(\theta(S_i\,,n))=B(\theta(S_i\,,n))-B(\theta(S_i\,,n-1))$, $B(\theta(S_i,n))$ is a Brownian motion white Gaussian noise generator with unit variance, and $\sigma'_{Ann}(\theta(S_i,n))$ is the annealing coefficient discrete derivative with respect to time increment ΔT between samples. Modulo 2π operations are performed after each phase update.

The gradient method for the weight updates is easier to implement than the adaptive array method. However, if it is desired to use the adaptive array method for optimizing the weights w_i [14-15] during the periods that the desired signal estimates are available, the pre weighted array estimates $\hat{x}_{i\,I,Q}(n)$, i = 1 to M have to be replaced by effective pre weighted estimates,

$\hat{x}_{i\ Eff\ I,Q}(n)=[\hat{\alpha}_{i\,I,Q}(n)+(1-\hat{\alpha}_{i\,I,Q}(n))/w_i(n)]\,\hat{x}_{i\,I,Q}(n)$ and by defining the vectors, (4.43)
$X_{Eff\ I,Q}=[\hat{x}_{1\ Eff\ I,Q}(n),\ \hat{x}_{2\ Eff\ I,Q}(n),\ldots,\ \hat{x}_{M\ Eff\ I,Q}(n)]$ with covariance matrix $R_{XX\,Eff}$,
$d_{I,Q}=\hat{s}_{D\,I,Q}(n)[1\,,1\,,\ldots,1]$ with M elements
where $\hat{s}_{D\,I,Q}(n)$ could be the estimated desired signal, the intrinsic identifying signal coding / transformation or the reference / wireless network identification sequences.

The minimum mean square error version of the weights is given by the following expression,

$$\left[w_1(n),w_2(n),\ldots,w_M(n)\right]_{MMSE}=\overline{X_{Eff}.d}\ R_{XX\,Eff}^{\ -1}\,, \quad (4.44)$$

Whereas, the MSINR (Maximum Signal to Interface and Noise Ratio) version of the weights becomes

$$\left[w_1(n),w_2(n),\ldots,w_M(n)\right]_{MSINR}=\overline{X_{Eff}.d}\ R_{I+N}^{\ -1}\text{, where }R_{I+N}=\left(X_{Eff}-d\ \overline{X_{Eff}.d}\right)\left(X-d\ \overline{X_{Eff}.d}\right) \quad (4.45)$$

Note that complex conjugate expressions are not used because the weights $w_i(t)$ are usually not separated into I (in phase) and Q (quadrature) components. As mentioned before, the ultra weak signal condition does not provide favourable means for implementing in phase and quadrature components before the array output. Therefore, multiple optimization procedures may have to be employed in order to optimize the weights $w_i(t)$ with respect to separate I and Q calculations.

Now, we will examine the nature of the injecting signals $S_i(t)$. The injecting signals are usually created by random number generators and also, the independence of the M injecting signals is a critical factor for the performance of the ASRA. Fortunately, the independence of the injecting signals is satisfied to a large extend by using stochastic annealing for the additive function parameter update.

In some implementations, the injecting signals $S_{i\,I,Q}(n)$ are not created by random number generators and simpler methods are used. For example, the base band signal $S_{i\,I,Q}(n)$ can be generated by using the data from the previous operation cycles and the required variance. When the probability distribution controlling variance $\sigma^2_{i\,I,Q}(S_{i\,I,Q},n)$ is updated, then by using the weighted Kernel method with K sampling points,

$$\sigma^2_{i\,I,Q}(S_{i\,I,Q},n) = \left[\sum_{J=0}^{K} S^2_{i\,I,Q}(n-j)\,Kern\left(\frac{S_{i\,I,Q}(n)-S_{i\,I,Q}(n-j)}{h_S}\right)\right]\cdot\left[\sum_{J=0}^{K} Kern\left(\frac{S_{i\,I,Q}(n)-S_{i\,I,Q}(n-j)}{h_S}\right)\right]^{-1} \quad (4.46)$$

where the smoothing parameter $h_S = 1.06\sigma^2(S_i).(K+1)^{-1/5}$ and $Kern(x) = Exp(-x^2/2)/\sqrt{2\pi}$ is the Gaussian Kernel, one can solve for $S_{i\,I,Q}(n)$. Another simpler method is to use the equal weight variance estimation on the K+1 samples,

$$S^2_{i\,I,Q}(n) = (K+1)(\sigma^2_{i\,I,Q}(S_{i\,I,Q},n) - \sigma^2_{i\,I,Q}(S_{i\,I,Q},n-1)) + S^2_{i\,I,Q}(n-K-1) \quad (4.47)$$

The sign of the signal is determined by examining the data from the previous operation cycles and the number of required zero crossings per time period T [38],

$$\overline{n_0(t)} = T.p(S_{i\,I,Q}(t)\,at\,S_{i\,I,Q}(t)=0).E\left\{\left|S'_{i\,I,Q}(t)\right| \mid S_{i\,I,Q}(t)=0\right\}$$, where $p(S_{i\,I,Q}(t))$ is the p.d.f. (4.48)

D. ASRA Performance

Let $\overline{S_i^2(t)} = A_i^2(S_i)\sigma_N^2$, where $\sigma_N^2 = \overline{N^2_{I,Q}(t)}$ is the preamplifier input noise variance and $A_i^2(S_i) \geq 1$. In fact, we are not able to generate and attenuate injecting signals $S_i(t)$, I = 1 to M to levels below the noise floor. Also, let the desired signal power $\overline{S_D^2(t)} = h_{SD}^2 = A^2(S_D)\sigma_N^2$, where $A^2(S_D) < 1$ (ultra weak signal condition), and the sum of the interfering signal power $\overline{S_{IN}^2(t)} = h_{IN}^2 = A^2(S_{IN})\sigma_N^2$. As mentioned before, $A_i(M)$ is the received signal $S_{R\,I,Q}(t)$ attenuation due to the creation of M arrays.

$$\begin{aligned}\overline{S_D(t).S_i(t)} &= \rho_i(S_D(t),S_i(t))\sqrt{\overline{S_D^2(t)}.\overline{S_i^2(t)}} \quad (4.49)\\ &= \rho_i(S_D(t),S_i(t))A_i(S_i)A(S_D)\sigma_N^2\end{aligned}$$

where the correlation coefficient range reduces to $0 < \rho(S_D(t),S_i(t)) < 1$ during acquisition and normal operation. Recall that the received signal is given by

$$S_{R\,I,Q}(t) = h_{SD}(t)s_{DI,Q}(t) + \sum_{K=1}^{Lc-1} h_K(t)s_{D\,I,Q}(t-kT) + h_{IN}(t)s_{IN\,I,Q}(t)$$

The pre weighted signal component power becomes

$$\begin{aligned}\overline{x^2_{i\,I,Q}(t)} &= \overline{\left[A_i(M).S_{R\,I,Q}(t) + S_{i\,I,Q}(t) + N_{i\,I,Q}(t)\right]^2} \quad (4.50)\\ &= \overline{\left[A_i(M).[h_{SD}(t)s_{DI,Q}(t) + \sum_{K=1}^{Lc-1} h_K(t)s_{D\,I,Q}(t-kT) + h_{IN}(t)s_{IN\,I,Q}(t)] + S_{i\,I,Q}(t) + N_{i\,I,Q}(t)\right]^2}\\ &\approx A_i^2(M)h_{SD}^2 + A_i^2(M)\sum_{K=1}^{Lc-1} h_K^2 + A_i^2(M)h_{IN}^2 + \overline{S_i^2(t)} + \sigma_N^2 + 2A_i(M)\overline{S_D(t).S_i(t)}\\ &+ 2A_i(M)\sum_{K=1}^{Lc-1} h_K\,\overline{s_D(t-kT).S_i(t)} + 2A_i(M)h_{IN}\,\overline{s_{IN}(t).S_i(t)}\end{aligned}$$

Using the following approximation for the weighted sum,

$$y_{T\,I,Q}(t) \approx \sum_{j=1}^{M} y_{j\,I,Q}(t) \approx \sum_{j=1}^{M} w_{j\,Eff}\,x_{j\,I,Q}(t) \approx \sum_{j=1}^{M} C\rho_i(S_D(t),S_j(t))x_{j\,I,Q}(t)$$, where C is a constant, (4.51)

$$SNR \approx \frac{\sum_{j=1}^{M}\{A_j^2(M)A^2(S_D)+2A_j(M)\rho_j(S_D(t),S_j(t))A_j(S_j)A(S_D)\}.C\rho_j(S_D(t),S_j(t)).\sigma_N^2}{\sum_{j=1}^{M} C\rho_j(S_D(t),S_j(t)).\sigma_N^2} \qquad (4.52)$$

$$\approx A_{avg}^2(M)A^2(S_D)+2A_{avg}(M)\rho_{avg}(S_D(t),S_j(t),M)A_{avg}(S_i)A(S_D)$$

$A_{avg}(M)$ is the average signal attenuation due to the creation of M conduction paths or array elements. $A_{avg}(S_i)$ is the average additive signal level in terms of multiples of noise level and $A(S_D)$ is the desired signal level in terms of fractions of the noise level. The first term is less than 1 and it can be ignored. $\rho_{avg}(S_D(t),S_j(t),M)$, which is the average correlation coefficient between the desired and additive signals is dependent on the possible variations of the desired signal, the array size M, and the interdependence of $S_j(t)$ (j = 1 to M) functions, and it increases as M becomes larger. The variations of the desired signal are dependent on the transmission and modulation schemes.

The condition for signal detection is $2A_{avg}(M)\rho_{avg}(S_D(t),S_j(t),M)A_{avg}(S_i)A(S_D)\geq 1$. (4.53)

Due to the interdependence of additive signals, for M>1

$$\rho_{avg}(S_D(t),S_j(t),M)\leq M.\rho_{avg}(S_D(t),S_j(t),1) \qquad (4.54)$$
$$=[1-\rho_{avg}(S_i(t),S_j(t),M,i\neq j)].M\rho_{avg}(S_D(t),S_j(t),1)$$

where $\rho_{avg}(S_i(t),S_j(t),M,i\neq j)$ is the average dependency of an additive signal with respect to the other M-1 additive signals, subject to the following condition,

$$[1-\rho_{avg}(S_i(t),S_j(t),M+1,i\neq j)](M+1)\geq[1-\rho_{avg}(S_i(t),S_j(t),M,i\neq j)]M\ , \qquad (4.55)$$

where the equality only occurs at $M_{\max}$.

Similarly, the expression for Signal to Interference and Noise Ratio becomes

$$SINR \approx \frac{\sum_{j=1}^{M}\{A_j^2(M)A^2(S_D)+2A_j(M)\rho_j(S_D(t),S_j(t))A_j(S_j)A(S_D)\}.C\rho_j(S_D(t),S_j(t)).\sigma_N^2}{\sum_{j=1}^{M}\{A_i^2(M)A^2(S_{IN})+A_i^2(S_i)+2A_i(M)\rho_j(S_{IN}(t),S_j(t))A_j(S_j)A(S_{IN})+1\}C\rho_j(S_D(t),S_j(t)).\sigma_N^2}$$

$$\approx \frac{A_{avg}^2(M)A^2(S_D)+2A_{avg}(M)\rho_{avg}(S_D(t),S_j(t),M)A_{avg}(S_j)A(S_D)}{A_{avg}^2(M)A^2(S_{IN})+A_{avg}^2(S_j)+2A_{avg}(M)\rho_{avg}(S_{IN}(t),S_j(t))A_{avg}(S_j)A(S_{IN})+1} \qquad (4.56)$$

Note that even though SINR is typically less than 1, the source separation module is able to extract the desired signal because SNR is slightly above 1. In spite of the advantages of processing the delayed contributions of the desired signal, the respective ISI terms have not been included in both the SNR and SINR analysis for simplicity.

V- Adaptive Transport Array (ATA) Preamplifier

Unlike the Adaptive Stochastic Resonance Array, the ATA method is a recent addition to the ultra weak signal pre amplifier technology. At this primitive stage, the ATA method can not be practically implemented in the compact wireless set and it requires further research for optimizing and reducing the number of arrays.

This section is basically an introduction to the preamplifier concept and in order to simplify the calculations, we have made the wild assumption that the aggregated sum of the received channel conditioned desired and interfered signal components does not contain any white noise component. The excess noise due to the received signal components will be considered in subsequent papers.

A. ATA Architecture

For the Ultra weak power condition, where the desired signal portion of the received signal has an average power below the noise power, we like to capture or emphasise the time periods that the instantaneous or short term noise power is smaller than or equal to the instantaneous received signal power in spite of the fact that the average noise power is greater the average signal power (see Appendix I). In the primary conduction path at the output of the nano antenna or nano cavity modules, if the ultra-weak signal condition still exists, the mentioned criteria only occurs for short periods of time.

However, if we have enough duplicates of the primary conduction path, then we can claim that most likely at any time instance, at least one of the duplicates will have a lower or equal instantaneous noise power with respect to the desired signal power. By emphasising or placing more weight on the special conduction path duplicate(s) that meet the mentioned requirement at each update period, we have effectively captured the higher signal to noise power signal for all time periods.

Therefore, an array is required and the conduction path segment $S_R(t)$ from the output of each antenna or the nano-cavity port is divided into M equal size transport paths $S_{Ri}(t)$, i=1 to M having the following base band format:

$$S_{Ri\,I}(t) = A_i(M)(S_{RI}(t)\cos\theta_i - S_{RQ}(t)\sin\theta_i), \tag{5.1}$$
$$S_{Ri\,Q}(t) = A_i(M)(S_{RI}(t)\sin\theta_i + S_{RQ}(t)\cos\theta_i) \text{ for i=1 to M}$$

The terms $A_i(M)$ and θ_i are respectively, the attenuation and the phase shift angle for each transport path due to the path division. $A_i(M)$ decreases as M increases and the phase shift is usually neglected up to the microwave range, but it may be considered for the millimeter wave bands. The mentioned terms can be determined from the geometry of the transport path division during actual implementation.

$$S_{Ri\,I}(t) = A_i(M)\, f_{MIX}\left(S_{D\,I}(t), S_{IN\,I}(t)\right) \tag{5.2}$$
$$= \mu_{i\,I}(t) \text{ with } \sigma_{iI}(t)Z_{iI}(t) \text{ as the corresponding noise}$$
$$S_{Ri\,Q}(t) = A_i(M)\, f_{MIX}\left(S_{D\,Q}(t), S_{IN\,Q}(t)\right)$$
$$= \mu_{i\,Q}(t) \text{ with } \sigma_{iQ}(t)Z_{iQ}(t) \text{ as the corresponding noise}$$

where $Z_{iI}(t)$ and $Z_{iQ}(t)$ are typically Gaussian white noise with unit variance.

The ultra weak signal condition of the transport paths does not allow for the creation of separate in-phase and quadrature components of the transport paths. The measurement access point is the output of the preamplifier. Therefore, separate I & Q calculations or measurements will be made and multiple optimizations will be employed. TheI & Q notations will be emitted for simplicity and the terminology $S_{Ri}(t) = \mu_i(t)$ with noise component $\sigma_i(t)Z_i(t)$ shall be used.

For each transport path, the terms $\sigma_i^2(t)$ and $\mu_i^2(t)$ represent the short term or instantaneous noise and signal powers, respectively. Each transport path with signal $S_{Ri}(t)$ will be multiplied by the emphasis or weight factor $w_i(t)$ for i = 1 to M at each update period. The weighted transport paths become $y_i(t)$ for i = 1 to M and the array output becomes $y_T(t) = \sum_{j=1}^{M} y_j(t)$ + measurement noise. The ATA output signal $y_T(t)$ becomes the detectable signal and therefore, it can be subjected to conventional signal processing. The multiplication shall be performed by a semi-ballistic nanostructure Gilbert cell such as nano FET Gilbert cell transistors with the minimal amount of field (voltage) to transport (current) translations. [18,19]

It is interesting to note that individual weighted transport paths can not be tapped for conventional processing, because the desired signal $h_{SD}(t)\, s_{DI,Q}(t)$ average power is below the average noise level and the only access point is the output of the array.

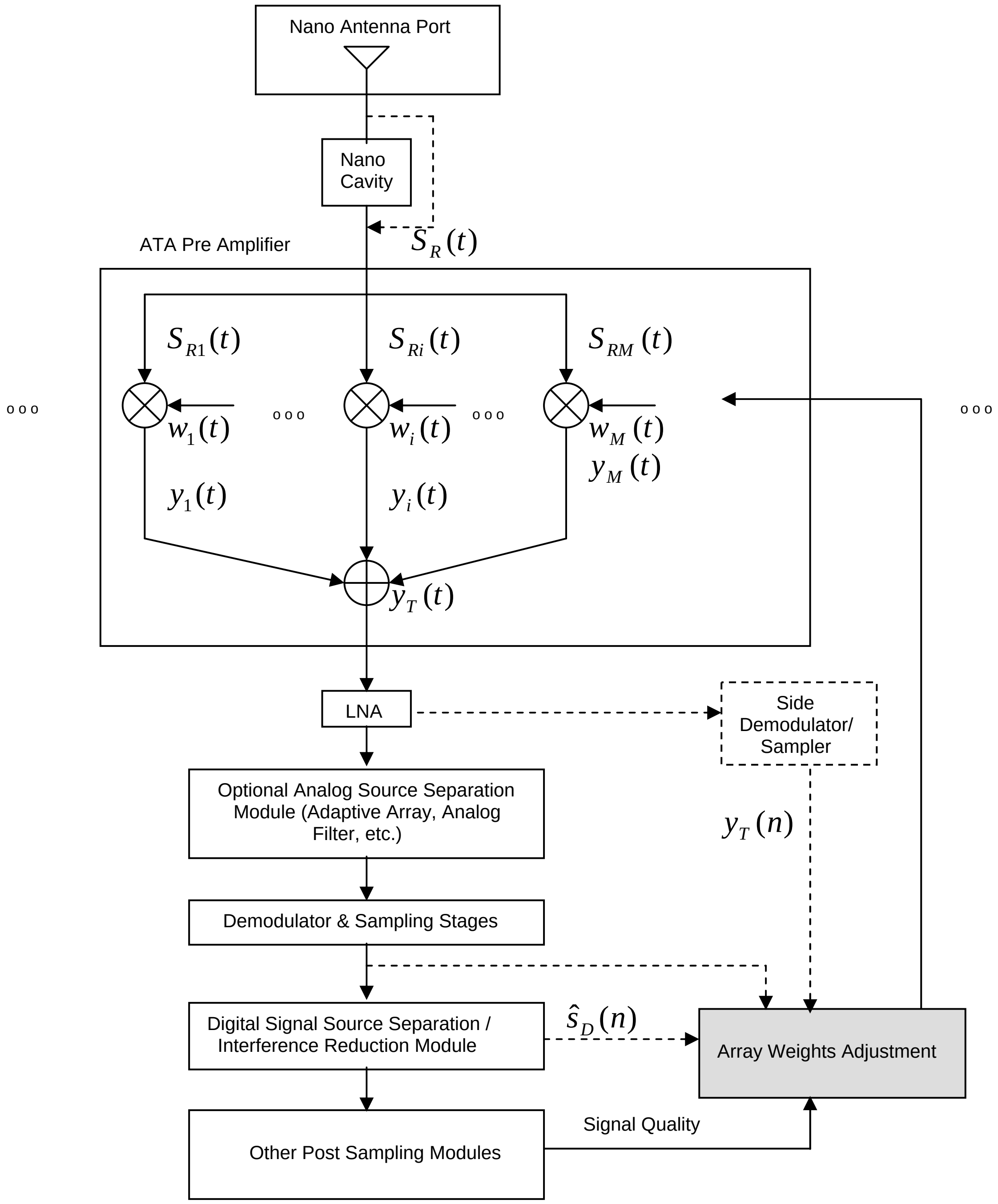

Fig. 3- Adaptive Transport Array

B. ATA Number of Required Paths

Before, concentrating on the weight assignment and the multiplication process, it is necessary to consider the number of required paths M for the ATA method. Assuming equal attenuation due to array creation $A = A_i(M)$ and by examining the instantaneous signal power $\mu_i^2(n)$ and noise power $\sigma_i^2(n)$ during cycle n operation for paths i =1 to M, we note that the noise power is dependent on the path, but the overall signal power is approximately the same for all paths. Therefore, generally $\mu_i^2(n) = \mu^2(n)$ and ignoring the cross correlation terms,

$$\mu^2(n) \approx A^2 \overline{f_{MIX}^2(t)}$$

$$\approx A^2\left(h_{SD}^2 + \sum_{K=1}^{Lc-1}[h_K^2 + 2h_K h_{SD} R_{SD,SD}(kT)] + h_{IN}^2 \right) \tag{5.3}$$

where short term expectation (covering cycle n operation) is applied over the mixed signal power. The power ratio for the desired and total received power is denoted by $\alpha_{D/MIX}^2 = \dfrac{h_{SD}^2}{f_{MIX}^2(t)}$

In order to detect the desired signal, the following condition should be met for instantaneous noise and signal powers,

$$h_{SD}^2 \geq \sigma_i^2(n) \text{ or } \frac{\alpha_{D/MIX}^2 \mu_i^2(n)}{A^2(M)} \geq \sigma_i^2(n) \tag{5.4}$$

where $\alpha_{D/MIX}^2$ is the power ratio between the desired and total received power. Recall that the ultra weak signal condition $\dfrac{\alpha_{D/MIX}^2 \overline{\mu_i^2(t)}}{A^2(M)} < \overline{\sigma_i^2(t)}$ for the average noise power.

The Beta distribution $\mathrm{B}(\sigma_i^2(t), \alpha_\mathrm{B}, \beta_\mathrm{B}, \sigma_{iMIN}^2(t), \sigma_{iMAX}^2(t))$ can be used to model the instantaneous noise term $\sigma_i^2(t)$ distribution with mean $\overline{\sigma_i^2(t)} = \sigma_{iMIN}^2(t) + \dfrac{(\sigma_{iMAX}^2(t) - \sigma_{iMIN}^2(t))\alpha_\mathrm{B}}{\alpha_\mathrm{B} + \beta_\mathrm{B}}$ and variance

$$\overline{\left(\sigma_i^2(t) - \overline{\sigma_i^2(t)}\right)^2} = \frac{(\sigma_{iMAX}^2(t) - \sigma_{iMIN}^2(t))^2 \alpha_\mathrm{B} \beta_\mathrm{B}}{(\alpha_\mathrm{B} + \beta_\mathrm{B})^2 (\alpha_\mathrm{B} + \beta_\mathrm{B} + 1)}$$

In order to meet the instantaneous power requirement for at least one array component at any time, we impose the condition

$$1/M \approx \Pr\left\{\sigma_i^2(t) \leq \frac{\alpha_{D/MIX}^2 \overline{\mu_i^2(t)}}{A^2(M)}\right\} \text{or } M \approx 2\left(1 - erf\,\frac{\overline{\sigma_i^2(t)} - \dfrac{\alpha_{D/MIX}^2 \overline{\mu_i^2(t)}}{A^2(M)}}{\sqrt{2}\, Dev(\sigma_i^2(t))}\right)^{-1} \tag{5.5}$$

where the equivalent normal distribution estimation of the Beta distribution has been used and

$$Dev(\sigma_i^2(t)) = \sqrt{\overline{\left(\sigma_i^2(t) - \overline{\sigma_i^2(t)}\right)^2}}$$

For M=100, $\quad \overline{\sigma_i^2(t)} - \dfrac{\alpha_{D/MIX}^2 \overline{\mu_i^2(t)}}{A^2(M)} \approx 1.65\sqrt{2}\, Dev(\sigma_i^2(t))$

C. ATA Weight Assignment

For the weighted path, the sampled transport path $S_{Ri}(n)$ is multiplied by the weight $w_i(n)$

$$y_i(n) = f_\mu(\mu, \sigma_i, w_i)\, w_i(n)\, \mu(n) + f_\sigma(\mu, \sigma_i, w_i)\, w_i(n) \sigma_i(n) Z_i(n) \text{ for i= 1 to M} \tag{5.6}$$
$$= \mu_{Li}(n) + \sigma_{Li}(n) Z_i(n), \text{ where } 1/w_i < f_\mu(\mu, \sigma_i, w_i) < 1 \text{ and } f_\sigma(\mu, \sigma_i, w_i) \approx 1$$

The multiplication reduction factor $f_\mu(\mu, \sigma_i, w_i)$ is dependent on the ratio $\dfrac{\mu^2}{\sigma_i^2}$, the strength of different signal components and the short term correlations of the signal components. Signal components can not be multiplied by w_i if their power levels are below σ_i^2. Even if the mentioned signals are aligned at certain time periods with power levels above σ_i^2, the aligned portions will be subject to multiplication by w_i without retaining the integrity of the individual signal components. Therefore, the desired signal will be multiplied by 1 instead of w_i at the output of the multiplier if its power level is below σ_i^2.

As mentioned before, the individual weighted paths are not accessible and our access point is the array output $y_T(n) = \sum_{j=1}^{M} y_j(n)$ + measurement error. Moreover, we are dealing with ultra low signal powers. Therefore, the Adaptive Array methods for optimizing the weights w_i [14-15] are not directly applicable. In order to use the method for weight assignment, a vector X will be defined as the input to the array,

$$X=\left[Af_{MIX}(n)f_{\mu 1}+\sigma_1 Z_1, Af_{MIX}(n)f_{\mu 2}+\sigma_2 Z_2,\ldots, Af_{MIX}(n)f_{\mu M}+\sigma_M Z_M\right]$$

where the effects of the multiplication reduction factors are included. Let $d=\hat{S}_D(n)[1,\ldots,1]$ as the desired signal estimate vector. The covariance matrix for the vector X becomes

$$R_{XX}\approx A^2\left(h_{SD}^2+\sum_{K=1}^{Lc-1}[h_K^2+2h_K h_{SD}R_{SD,SD}(kT)]+h_{IN}^2\right)\begin{bmatrix} f_{\mu 1}^2 & f_{\mu 1}f_{\mu 2} & . & . & f_{\mu 1}f_{\mu M} \\ f_{\mu 2}f_{\mu 1} & f_{\mu 2}^2 & . & . & f_{\mu 2}f_{\mu M} \\ . & . & . & . & . \\ . & . & . & . & . \\ f_{\mu M}f_{\mu 1} & . & . & . & f_{\mu M}^2 \end{bmatrix} \tag{5.7}$$

$$+\begin{bmatrix} \sigma_1^2 & 0 & 0 & . & 0 \\ 0 & \sigma_2^2 & 0 & . & 0 \\ 0 & 0 & \sigma_2^2 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & \sigma_M^2 \end{bmatrix}$$

The minimum mean square error (MMSE) version of the weight vector becomes

$$\begin{aligned} w_{MMSE}(n) &= \left[w_1(n), w_2(n),\ldots, w_M(n)\right]_{MMSE} \\ &= \overline{X.d}\ R_{XX}^{\ -1} \end{aligned}$$

The MSINR (Maximum Signal to Interface and Noise Ratio) version of the weight vector becomes

$$\begin{aligned} w_{MSINR}(n) &= \left[w_1(n), w_2(n),\ldots, w_M(n)\right]_{MSINR} \\ &= \overline{X.d}\ R_{I+N}^{\ -1} \text{ and} \end{aligned}$$

$$R_{I+N}=\overline{\left(X-d\ \overline{X.d}\right)\left(X-d\ \overline{X.d}\right)} \tag{5.8}$$

$$\approx A^2 h_{IN}^2\begin{bmatrix} f_{\mu 1}^2 & f_{\mu 1}f_{\mu 2} & . & . & f_{\mu 1}f_{\mu M} \\ f_{\mu 2}f_{\mu 1} & f_{\mu 2}^2 & . & . & f_{\mu 2}f_{\mu M} \\ . & . & . & . & . \\ . & . & . & . & . \\ f_{\mu M}f_{\mu 1} & . & . & . & f_{\mu M}^2 \end{bmatrix}+\begin{bmatrix} \sigma_1^2 & 0 & 0 & . & 0 \\ 0 & \sigma_2^2 & 0 & . & 0 \\ 0 & 0 & \sigma_2^2 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & \sigma_M^2 \end{bmatrix}$$

As mentioned before, the desired signal can only be amplified if the noise power becomes lower than the desired signal power and due to the limitations on the array size, only one path will most likely satisfy the mentioned low noise power requirement. Moreover, due to the instantaneous noise fluctuations, we have to predict the path instantaneous noise power for the next update period in order to assign the weights. Therefore, the Adaptive Array method does not seem to be practical for the ATA preamplifier and another policy is required.

Let w_{SD} be the minimum weight that is required for the paths that satisfy the $\frac{\alpha_{D/MIX}^2\mu_i^2(n)}{A^2(M)}\geq\sigma_i^2(n)$ requirement. A good policy would be to assign the weight $w_i>w_{SD}$ for transport paths which satisfy the $\frac{\alpha_{D/MIX}^2\mu_i^2(n)}{A^2(M)}\geq\sigma_i^2(n)$ requirement for the next update period and to assign $w_i=1$ for other cases. Note that if we set $w_i>1$ for cases where the noise power is larger than the desired signal power, the signal component would not be amplified, but the noise component will be amplified. In order to estimate w_{SD}, we assume that only one of the paths satisfies the above signal requirement and M-1 paths are multiplied by 1.

The desired signal portion of the array output is equated to the output array noise and the result becomes:

$$w_{SD} = M\left[\frac{\overline{\sigma^2(t)}}{A^2 h_{SD}^2} - 1\right] + 1, \text{ where } \overline{\sigma^2(t)} = \overline{\sigma_i^2(t)} \text{ for i = 1 to M} \quad (5.9)$$

The following issues have to be considered for the weight assignment:

1- During an update period, the noise and signal powers for each path have to be predicted for the next update period.

2- The desired signal power is not know, especially at the start of the tracking mode.

The path instantaneous signal and noise powers have to be predicted for the next update period in order to assign the weights. Due to the use of semi-ballistic nano structures, it can be shown that the for any level of signal power, the noise statistics are basically pseudo random sequences which can be retrieved by pseudo random noise generating state machine (see Appendix I). There are various means of collecting statistics for building the state machine. The array output is fully accessible and its instantaneous signal and noise powers can be calculated by methods of stochastic calculus.

At the start of the tracking phase, by setting the weight for one of the paths to a low value of $w > 1$ and the rest of the weights to 1 and retaining the weight assignment for many cycles, eventually the output signal power is maximized. In effect, we are simultaneously assuming a large power threshold $\alpha_{D/MIX}^2 \mu_i^2(n) / A(M)$ for detecting the desired signal. If a minimum level of desired signal quality (metric) is not detected by the post sampling modules, then the threshold is lowered by increasing the weight for the particular path until the desired signal is detected.

By repeating this procedure for other paths, the statistics of instantaneous noise power $\sigma_i^2(t)$ will be approximated for the different cases of $\mu_i^2(n) \approx \mu^2(n)$ signal powers. The Ultra weak signal channel environment is inherently time invariant, therefore the pseudo random noise state model has to incorporate the different signal power $\frac{\mu_i^2(n)}{\sigma_i^2(n)}$ conditions. Also, by performing other permutations such as setting 2 or more of the weights w_j to common values above 1, the approximate multiplication reduction factors $f_\mu(\mu, \sigma_i, w_i)$ can be retrieved as a function of $\frac{\mu^2}{\sigma_i^2}$.

The assignment of $w_i \geq w_{SD}$ for transport paths which satisfy the desired signal power threshold and the assignment of $w_i = 1$ for other cases is optimum only if $\frac{\mu^2}{\sigma_i^2}$ is predicted exactly. As indicated in the next section, the weights for the operating cycle n will be assigned during n-1 operating cycle by using the a priori estimates $\tilde{\sigma}_i^2(n)$ and $\tilde{\mu}_i^2(n)$. During n-1 operating cycle, a priori estimates $\tilde{\sigma}_i^2(n-1)$, $\tilde{\mu}_i^2(n-1)$ and a posteriori estimates $\hat{\sigma}_i^2(n-1)$ and $\hat{\mu}_i^2(n-1)$ are also available. Therefore, the following policy for the weight assignment can be used:

1- Assign the weight $w_i \geq w_{SD}$ for paths that satisfy the condition $\frac{\alpha_{D/MIX}^2 \tilde{\mu}_i^2(n)}{A^2(M)} \geq \tilde{\sigma}_i^2(n)$

2- Assign the weights $w_{SD} > w_i > 1$ in a linear fashion for paths that satisfy the criteria

$$\frac{\alpha_{D/MIX}^2 \tilde{\mu}_i^2(n)}{A^2(M)} < \tilde{\sigma}_i^2(n) \leq \frac{\alpha_{D/MIX}^2 \tilde{\mu}_i^2(n)}{A^2(M)}[1 + \gamma(\sigma_i^2(n), \mu_i^2(n))] \quad (5.10)$$

where $\gamma(\sigma_i^2(n), \mu_i^2(n)) = \left|\frac{\tilde{\sigma}_i^2(n-1)}{\tilde{\mu}_i^2(n-1)} - \frac{\hat{\sigma}_i^2(n-1)}{\hat{\mu}_i^2(n-1)}\right| \cdot \left[\frac{\tilde{\sigma}_i^2(n)}{\tilde{\mu}_i^2(n)}\right]^{-1}$, 3- Assign the weight $w_i = 1$ for paths that satisfy the condition $\tilde{\sigma}_i^2(n) \geq \frac{\alpha_{D/MIX}^2 \tilde{\mu}_i^2(n)}{A^2(M)}[1+\gamma(\sigma_i^2(n), \mu_i^2(n))]$

D. Transport Path Base-band Signal Estimation & ATA Cycle Operations

The instantaneous signal and noise powers can be estimated by using the array output measurement and the method of stochastic calculus. The stochastic differential expressions for the transport paths, weighted transport paths i (i = 1 to M) and array output become

$$
\begin{aligned}
S_{Ri}(t)dt + \sigma_i(t)Z_i(t)dt &= \mu_i(t)dt + \sigma_i(t)dB_i(t) \\
&= dL_i(t) \\
y_i(t)dt &= \mu_{Li}(t)dt + \sigma_{Li}(t)dB_i(t) \\
&= dL_{wi}(t) \\
&\approx f_{\mu_i}.w_i(t)\mu_i(t)dt + w_i(t)\sigma_i(t)dB_i(t) \\
y_T(t)dt &= \mu_T(t)dt + \sum_{j=1}^{M}\sigma_{Lj}(t)dB_j(t) \\
&= \mu_T(t)dt + \sigma_T(t)dB(t) \\
&= dL_T(t), \text{ where } \sigma_T(t) = \left[\sum_{j=1}^{M}\sigma_{Lj}^2(t)\right]^{1/2}
\end{aligned}
\tag{5.11}
$$

The terms μ and σ^2 are the drift (instantaneous mean) and diffusion (instantaneous variance) for the respective stochastic process. The term $dB_i(t)$ is the brown motion or Wiener process with variance $\overline{d^2B_i(t)} = dt$. The drift and diffusion coefficients can be estimated by the work of Ait-Sahalia, Jiang, Knight, Stanton, Chapman and Pearson and the relevant procedures are outlined in Ref. [39-41].

If the sampled base band version of the weighted path $y_i(t)$, i.e. $y_i(n)$ were available, then by using numerical Integration methods such as Trapezoid Law $L_{wi}(n-j) = 0.5\Delta T\left(y_i(n-j) + y_i(n-j-1)\right) + L_{wi}(n-j-2)$, the drift and diffusion parameters $\mu_{Li}(n)$ and $\sigma_{Li}^2(n)$ of the weighted paths could have been estimated by the following kernel weighted equations:

$$
\mu_{Li}(n) = \frac{\sum_{J=0}^{K}\left[L_{wi}(n-j) - L_{wi}(n-j-1)\right] Kern\left[\frac{L_{wi}(n) - L_{wi}(n-j-1)}{h_S}\right]}{\Delta T \sum_{J=0}^{K} Kern\left[\frac{L_{wi}(n) - L_{wi}(n-j-1)}{h_S}\right]}
\tag{5.12}
$$

$$
\sigma_{Li}^2(n) = \frac{\sum_{J=0}^{K}\left[L_{wi}(n-j) - L_{wi}(n-j-1)\right]^2 Kern\left[\frac{L_{wi}(n) - L_{wi}(n-j-1)}{h_S}\right]}{\Delta T \sum_{J=0}^{K} Kern\left[\frac{L_{wi}(n) - L_{wi}(n-j-1)}{h_S}\right]}
$$

where K+1 sampling points have been considered . The smoothing parameter $h_S = 1.06\,\text{var}(L_{wi}).(K+1)^{-1/5}$ and the $Kern(x) = Exp(-x^2/2)/\sqrt{2\pi}$ is the Gaussian Kernel. By knowing the drift and diffusion coefficients of the weighted paths and the weights $w_i(n)$, the instantaneous noise variance of the paths $\sigma_i^2(n)$ and the effective instantaneous path signal power $\left(f_{\mu_i}.\mu_i(n)\right)^2$ could be retrieved. After the measurement of array output, a posteriori estimates of the weighted transport paths shall be used for the mentioned drift and diffusion calculations.

The Milstein method [34] for estimating $L_{wi}(n)$ (or $y_i(n)$) from $L_{wi}(n-1)$ (or $y_i(n-1)$) is indicated below:

$$L_{wi}(n) = L_{wi}(n-1) + \mu_{Li}(n)\Delta T + \sigma_{Li}(n).\Delta B_i(n-1) + 0.5\sigma_{Li}(n)\sigma'_{Li}(n)\left(\Delta B_i^2(n-1) - \Delta T\right) \quad (5.13)$$

$$= L_{wi}(n-1) + f_{\mu_i}.w_i(n)\mu_i(n)\Delta T + w_i(n)\sigma_{Li}(n).\Delta B_i(n-1)$$

$$+ 0.5w_i(n)\sigma_i(n)(w_i(n)\sigma'_i(n) + w'_i(n)\sigma_i(n))\left(\Delta B_i^2(n-1) - \Delta T\right)$$

The Brownian increments $\Delta B_i(n-1)$ are nearly independent with variance $\overline{(\Delta B_i)^2} = \Delta T$

During the (n-1) period, the a priori estimates for the next period (n) which include the drift $\tilde{\mu}_i(n)$, the diffusion $\tilde{\sigma}_i^2(n)$, the Brownian increment $\Delta\tilde{B}_i(n-1)$, and the multiplication reduction factor $\tilde{f}_{\mu_i}$ can be determined by using either simple predictors at the start of the operation or by using the pseudo random noise Markovian state models [43-44] (see Appendix I). The weights $w_i(n)$ are also assigned at this stage.

The method for estimating the base band signals of the transport paths by measuring the array output is presented here. After making the measurement from the output of the array $y_T(n)$ or equivalently $L_T(n)$ due to $y_T(t) = \dfrac{dL_T(t)}{dt}$, the a posteriori estimates for the above terms can be determined. For this purpose, the Kalman filtering method [37] will be used.

For the Kalman state equation, we regroup $L_{wi}(n)$, as follows

$$L_{wi}(n) = L_{wi}(n-1) + u_i(n-1) + e_i(n-1) \text{ for i =1 to M} \quad (5.14)$$

Where $L_{wi}(n)$ and $L_{wi}(n-1)$ are the current and previous Kalman process states, respectively. The term $u_i(n-1)$ is the driving function and $e_i(n-1)$ is the process noise with variance $Q_i(n) = \overline{e_i^2(n-1)}$.

$$u_i(n-1) = \tilde{f}_{\mu_i}.w_i(n)\tilde{\mu}_i(n)\Delta T + w_i(n)\tilde{\sigma}_i(n).\Delta\tilde{B}_i(n-1) + \quad (5.15)$$

$$+ 0.5w_i(n)\tilde{\sigma}_i(n)(w_i(n)\tilde{\sigma}'_i(n) + w'_i(n)\tilde{\sigma}_i(n))\left(\Delta\tilde{B}_i^2(n-1) - \Delta T\right)$$

$$e_i(n-1) = [f_{\mu_i}.w_i(n)\mu_i(t) - \tilde{f}_{\mu_i}.w_i(n)\tilde{\mu}_i(n)]\Delta T + w_i(n)[\sigma_i(n).\Delta B_i(n-1) - \tilde{\sigma}_i(n).\Delta\tilde{B}_i(n-1)]$$

$$+ 0.5w_i(n)\sigma_i(n)(w_i(n)\sigma'_i(n) + w'_i(n)\sigma_i(n))\left(\Delta B_i^2(n-1) - \Delta T\right)$$

$$- 0.5w_i(n)\tilde{\sigma}_i(n)(w_i(n)\tilde{\sigma}'_i(n) + w'_i(n)\tilde{\sigma}_i(n))\left(\Delta\tilde{B}_i^2(n-1) - \Delta T\right)$$

For the Kalman measurement equation, the array output will be used, as follows:

$L_T(n) = \sum_{j=1}^{M} L_{wj}(n) + v_L(n)$ where $v_L(n)$ is the zero mean measurement noise with variance $R(n) = \overline{v_L^2(n)}$

By using the a posteriori estimates $\hat{L}_{wi}(n-1)$ of the (n-1) cycle, we obtain the following a priori estimates (or state project):

$$\tilde{L}_{wi}(n) = \hat{L}_{wi}(n-1) + u_i(n-1) \text{ for i = 1 to M,} \quad (5.16)$$

$\tilde{e}_i = L_{wi}(n) - \tilde{L}_{wi}(n)$ is the a priori estimate error with variance $\tilde{P}_i(n) = \overline{\tilde{e}_i^2(n)}$

$\hat{e}_i = L_{wi}(n) - \hat{L}_{wi}(n)$ is the a posteriori estimate error with variance $\hat{P}_i(n) = \overline{\hat{e}_i^2(n)}$

The index n in the variance expressions $Q_i(n)$, $R(n)$, $\tilde{P}_i(n)$ and $\hat{P}_i(n)$ implies that the variance estimates were obtained by using all of the available data up to the nth period.

$$\tilde{P}_i(n) = \hat{P}_i(n-1) + Q_i(n-1) \text{ is the error variance projection} \quad (5.17)$$

$K_i(n) = \dfrac{\tilde{P}_i(n)}{\sum_{j=1}^{M}\tilde{P}_j(n) + R(n-1)}$ is the Kalman gain

After receiving the demodulated and sampled output of the array $Y_T(n)$ (or equivalently $L_T(n)$), the a posteriori estimates of the weighted path $\hat{L}_{wi}(n)$ become

$$\hat{L}_{wi}(n) = \tilde{L}_{wi}(n) + K_i(n)(L_T(n) - \sum_{j=1}^{M}\tilde{L}_{wj}(n)) \text{, i = 1 to M} \tag{5.18}$$

$$= \tilde{L}_{wi}(n) + \frac{\hat{P}_i(n-1) + Q_i(n-1)}{\sum_{j=1}^{M}(\hat{P}_i(n-1) + Q_i(n-1)) + R(n-1)}(L_T(n) - \sum_{j=1}^{M}\tilde{L}_{wj}(n))$$

$\hat{P}_i(n) = (1 - K_i(n))\tilde{P}_i(n)$ for the updated variance of the a posteriori estimate error of the weighted paths.

By the knowledge of a posteriori weighted path estimates $\hat{L}_{wi}(n)$, the estimated drift $\hat{\mu}_{Li}(n)$ and diffusion $\hat{\sigma}_{Li}^2(n)$ can be determined by the weighted kernel method of equation 5.12. Since the weights $w_i(n)$ are known, the a posteriori estimates for drift $\hat{\mu}_i(n)$ and the product of drift and the multiplication factor $\hat{f}_{\mu_i}\hat{\mu}_i(n)$ can be easily determined by simple division of $\hat{\mu}_{Li}(n)$ and $\hat{\sigma}_i(n)$ by the weights $w_i(n)$.

Then, the process noise estimates can be retrieved by the difference between the a priori and a posteriori weighted path estimates.

$\hat{e}_i(n-1) = \hat{L}_{wi}(n) - \tilde{L}_{wi}(n)$, which can be used to update the process noise variance $Q_i(n)$. (5.19)

The a posteriori estimates of the Brownian increments $\Delta\hat{B}_i(n-1)$ can be retrieved by using the process noise estimates in quadratic format, as follows:

$$\begin{aligned}\hat{e}_i(n-1) &= [\hat{f}_{\mu_i}.w_i(n)\hat{\mu}_i(t) - \tilde{f}_{\mu_i}.w_i(n)\tilde{\mu}_i(n)]\Delta T + w_i(n)[\hat{\sigma}_i(n).\Delta\hat{B}_i(n-1) - \tilde{\sigma}_i(n).\Delta\tilde{B}_i(n-1)] \\ &+ 0.5w_i(n)\hat{\sigma}_i(n)(w_i(n)\hat{\sigma}_i'(n) + w_i'(n)\hat{\sigma}_i(n))\left(\Delta\hat{B}_i^2(n-1) - \Delta T\right) \\ &- 0.5w_i(n)\tilde{\sigma}_i(n)(w_i(n)\tilde{\sigma}_i'(n) + w_i'(n)\tilde{\sigma}_i(n))\left(\Delta\tilde{B}_i^2(n-1) - \Delta T\right)\end{aligned} \tag{5.20}$$

The mentioned a posteriori estimates and the a priori estimates will be used to update the drift and diffusion predictors and the pseudo noise state model. The latest estimate of the measurement noise $\hat{v}_L(n)$ becomes:

$$\hat{v}_L(n) = L_T(n) - \sum_{j=1}^{M}\hat{L}_{wj}(n) \tag{5.21}$$

The measurement variance $R(n)$ of the array output will be updated by using the measurement noise estimate at the nth cycle. The block diagram for ATA weight adjustment and update process is shown in Fig 4.

VI- CONCLUSION

Ultra weak signal processing is required for recovering communication signals that have power levels below the noise floor of the receiver. The concepts can be used for recovering the ultra weak signals for both wireless and wired media. The signal processing is fundamental in implementing the long awaited Source Separation technology in the receiver modules for the sake of increasing the user capacity. It also facilitates the deployment of wireless sensors and actuators for many applications including remote operations. It is interesting to note that the ultra weak preamplifiers do not require extra bandwidth or transmission protocol modifications for their operation.

Currently, the Adaptive Stochastic Resonance Array (ASRA) is the most flexible technology for implementing the ultra weak wireless preamplifier module. The preamplifier does not require the internal signal dynamics for its operation. The array has been successfully implemented for the base band version as Supra-threshold Stochastic Resonance [16, 17] and its infrastructure is well known. The structure for the additive signals which is dependent on the transmission protocol can be improved as better algorithms become available. In fact, if programmable module is used for the preamplifier section, the algorithm modifications for the additive signals, quality measurement and the adaptive parameter updates can be loaded for improved performance. The excess interference due to the additive signals can be readily eliminated in the source separation module because the additive signals and their contribution to the array output are known.

The ultra weak signal processing provides a dynamic jump in the market for massive communication services. Moreover, by resolving the signal detection and radio frequency power overloads, there will be a huge market for the inclusion of wireless modules in almost all of the electronic and electromechanical equipment. Also, the remote operation facility provided by the signal processing leads to dynamic improvements in production, management, service offerings, diversified remote inspections, self employment and other areas which lead to economic growth.

During the (n-1) operation cycle, calculate the (n) cycle path a priori estimates for drift $\tilde{\mu}_i(n)$, diffusion $\tilde{\sigma}_i(n)$, Brownian increment $\Delta\tilde{B}_i(n-1)$ and multiplication reduction factor $\tilde{f}_{\mu_i}$ by using the (n-1) cycle posteriori estimates, simple predictors, random noise observable Markovian state machine, paths i=1 to M

↓

During the (n-1) operation cycle, estimate the Kalman Filter driving function $u_i(n-1)$, the path weights $w_i(n)$ for the (n) operation cycle, i = 1 to M

↓

During the (n-1) operation cycle, calculate the weighted paths a posteriori estimates $\tilde{L}_{wi}(n)$ for the nth cycle by using the driving function and the a posteriori estimates $\hat{L}_{wi}(n-1)$

↓

During the (n) operation cycle, by using the output of the array, calculate the Kalman filter gain and the a posteriori estimates $\hat{L}_{wi}(n)$ for the weighted paths and update the Kalman filter parameters

↓

By using the a posteriori estimates $\hat{L}_{wi}(n)$ for the weighted paths, the (n) cycle a posteriori weighted paths drift $\hat{\mu}_{Li}(n)$ and diffusion $\hat{\sigma}^2_{Li}(n)$ terms can be determined by the methods of stochastic calculus, such as the weighted Gaussian kernel method

↓

Estimate the (n) cycle a posteriori drift $\hat{\mu}_i(n)$, diffusion $\hat{\sigma}_i^2(n)$, multiplication factor $\hat{f}_{\mu_i}$, and the Brownian increments $\Delta\hat{B}_i(n-1)$ for paths i = 1 to M, update the array output measurement noise $R(n)$ and weighted path estimate error $Q_i(n)$ statistics, and update the pseudo noise Markovian state machine

Fig. 4- Adaptive Transport Array Update Process

Appendix I- Electronic Transport Mechanism

Boltzmann transport equation [45-48] is used for examining the receiver transport and noise mechanism due to its simple structure. However, for the instantaneous noise power analysis and its pseudo randomness, a simpler method that is based on stochastic calculus will be employed.

$$\frac{d\vec{x}}{dt} \equiv \vec{v}(\vec{k}) = \frac{1}{\hbar}\nabla_k \varepsilon(\vec{k}), \qquad \text{(I.1)}$$

where $\vec{x}$ is the electron position, $\vec{v}$ is the electron drift velocity, $\vec{k}$ is the wave vector, $\varepsilon(\vec{k})$ is the energy vector,

$$\vec{\mathrm{E}}_{Eff}(\vec{x},t) = \vec{\mathrm{E}}_{Int}(\vec{x},t) + \vec{\mathrm{E}}_{Ext}(V_{bias}, f_{MIX}(S_D(t), S_{IN}(t)) \qquad \text{(I.2)}$$

where the internal field $\vec{\mathrm{E}}_{Int}(\vec{x},t)$ is generated by internal forces and density profiles

$$\frac{d\vec{\rho}}{dt} = \hbar\frac{d\vec{k}}{dt} = -q\vec{\mathrm{E}}_{Eff} + \vec{F}_r\,, \tag{I.3}$$

where $\vec{\rho}$ is the momentum, $\vec{\mathrm{E}}_{Eff}$ is the effective electric field, $\vec{F}_r$ is the random impulse force on electron due to scattering and it is given by

$$\vec{F}_r = \sum_i \hbar\vec{u}_i\delta(t-t_i) = \overline{\{\vec{F}_r\}} + \vec{F}_r^{\,0}\,, \tag{I.4}$$

where $\overline{\{\vec{F}_r\}}$ is the momentum driven drag force and $\vec{F}_r^{\,0}$ is the zero mean fluctuating force.

$$\overline{\{\vec{F}_r\}} = \int \vec{u}W(\vec{k},\vec{k}+\vec{u})d\vec{u} \approx -\lambda(0)m_e\vec{v} \text{ and} \tag{I.5}$$

$$\vec{F}_r^{\,0} \approx \sigma(\vec{F}_r)\frac{dB_t}{dt}$$

where B_t is a zero mean Wiener process, $\sigma^2(\vec{F}_r)$ is the variance of the random scattering force on the electron, $\lambda(\vec{k})$ is the scattering rate for the electron wave vector $(\vec{k})$, and $W(\vec{k},\vec{k}')$ is the transition rate satisfying $\lambda(\vec{k}) = \int W(\vec{k},\vec{k}')d\vec{k}'$, $\lambda(0) = 1/\tau_f$, where τ_f is the average time between collisions

Therefore,

$$\vec{F}_r = -\lambda(0)m_e\vec{v} + \sigma(\vec{F}_r)\frac{dB_t}{dt} \tag{I.6}$$

$\lambda(\vec{k})\Delta t$ is the probability that a jump in momentum will occur in a small time interval Δt and if a scattering event has occurred at time t_i, $\vec{k}_i = \vec{k}(t_i^-)$ and $\vec{k}_i + \vec{u}_i = \vec{k}(t_i^+)$, then the probability distribution function for the amplitude of the jump would be $\rho_k(\vec{u}_i) = \dfrac{W(\vec{k}_i,\vec{k}_i+\vec{u}_i)}{\lambda(\vec{k}_i)}$

Note that for a conduction path, there is a correspondence between the scatterings and a Markovian state transition probability model. The momentum jumps in a given time frame indicate transitions into new states. For semi-ballistic transport, the number of Markovian states is limited, because the scattering rate is lower with respect to non-ballistic transport and depending on the relaxation time period, there is a tendency for electron distribution to return to equilibrium.

Even though there is a weak dependency between the signal power and the noise power transitions, it is customary to model one state machine for a range of average signal power to noise power ratio. Let $q:\{Sp_1, Sp_2,..., Sp_L\}$ be the L Markovian states for a noise analysis of a simple transport path for a known range of signal to noise power ratio and let $\{\alpha_1(\overline{\sigma_N^2}), \alpha_2(\overline{\sigma_N^2}), \ldots, \alpha_V(\overline{\sigma_N^2})\}$ be the V possible noise power symbols which are normalized with respect to the average noise power $\overline{\sigma_N^2}$.

The Markovian state machine [43,44] is modelled by optimizing the observable symbol probability distributions, $b_j(k) = \Pr\{\alpha_k(\overline{\sigma^2})\,at\,time\;period\;n|\,q(n) = Sp_j\}$ for j = 1 to L and k = 1 to V and the state transition distributions $a_{ij}(k) = \Pr\{q(n+1) = Sp_j \mid q(n) = Sp_i\}$ for i, j = 1 to L. In practice, the Markovian state machine is constructed by indirect noise power measurement during each measurement sampling period. Therefore, the Markovian transitions are modelled discretely with the same time frame as the measurement sampling period. If the measurement sampling period is $\Delta\mathrm{T}$, the probability of electron scattering in time period $\Delta\mathrm{T}$ is specified below,

$$\Pr\{(t_i - t_{i-1}) \le \Delta\mathrm{T}\} = 1 - \exp\{-\int_{t_{i-1}}^{t_{i-1+\Delta\mathrm{T}}} \lambda(\vec{k}(t'))dt'\}. \tag{I.7}$$

The Boltzmann Transport Equation (BTE) is given by the expression,

$$\frac{df(\vec{x},\vec{k})}{dt} + \vec{v}(\vec{k}).\nabla_X f(\vec{x},\vec{k}) - \frac{q}{\hbar}\vec{\mathrm{E}}_{Eff}(\vec{x},t).\nabla_k f(\vec{x},\vec{k}) = \int f(\vec{x},\vec{k}')W(\vec{k}',\vec{k})d\vec{k}' - \lambda(\vec{k})f(\vec{x},\vec{k}) \tag{I.8}$$

$$= 1/2\sum_{i,j=1}^{3}\frac{\partial^2}{\partial k_i \partial k_j}[\sigma_{ij}(\vec{k})f(\vec{x},\vec{k})] - \frac{1}{\hbar}\nabla_k.[E\{\vec{F}_r\}f(\vec{x},\vec{k})]$$

where $f(\vec{x},\vec{k})$ is the electron distribution function, q is the electronic charge and $\sigma_{ij}(\vec{k}) = \int \vec{u}_i \vec{u}_j W(\vec{k}_i, \vec{k}_i + \vec{u}_i) d\vec{u}_i$

The current longitudinal noise auto covariance function can be retrieved as the transient solution of BTE, subject to the following special initial condition,

$$f(\vec{x},\vec{k},t)\Big|_{t=0} = (\vec{v}(\vec{k}) - <\vec{v}>_X) f_{SS}(\vec{x},\vec{k}), \tag{I.9}$$

where $f_{SS}(\vec{x},\vec{k})$ is the steady state solution of the electron distribution function and $<\vec{v}>_X \equiv \int \vec{v}(\vec{k}) f_{SS}(\vec{x},\vec{k}) d\vec{k}$.

Typically, the Fermi distribution is used for the steady state electron distribution,

$f_{SS}(\vec{x},\vec{k}) = [\exp(\varepsilon(\vec{k}) - \mu_f)/K_B T + 1]^{-1}$, where K_B is the celebrated Boltzmann constant and T is the Temperature.

The current noise auto covariance function becomes

$K_J(\vec{x},t) \propto q^2 \int \vec{v}(\vec{k}) f(\vec{x},\vec{k},t) d\vec{k}$, $t \geq 0$

$\Phi_J(\vec{x},w) \propto \int K_J(\vec{x},\tau) e^{jw\tau} d\tau$ is the current density noise power spectral density with the average noise power of $\Phi_J(\vec{x},0)$.

For ballistic transport, the average current noise power at room temperature is given by $8K_B T q^2 .T_{TR} / h$, where T_{TR} is the ballistic net transmission coefficient. For semi-ballistic transport, there is a partial contribution of the non-ballistic current noise power density of $\dfrac{4K_B T.L}{(1 + w^2 \tau_f^2) n q \mu_e A}$ where $\tau_f^2 = 1/\lambda^2(0)$ is the square of the average time between collisions (relaxation time), A is the cross sectional area of the conduction path, L is the length of the conduction path and μ_e is the electron mobility. The average current noise power for non-ballistic transport path is $\dfrac{4K_B T.L}{n q \mu_e A}$

For the instantaneous noise power analysis, the force equation is transformed into stochastic differential equation,

$$\frac{d\vec{\rho}}{dt} = \hbar \frac{d\vec{k}}{dt} = m_e \frac{d\vec{v}(\vec{k})}{dt} = -q\vec{\mathrm{E}}_{Eff} + \vec{F}_r = -q\vec{\mathrm{E}}_{Eff} - \lambda(0) m_e \vec{v} + \sigma(\vec{F}_r) \frac{dB_t}{dt} \tag{I.10}$$

$$m_e d\vec{v} = (-q\vec{\mathrm{E}}_{Eff} - \lambda(0) m_e \vec{v}) dt + \sigma(\vec{F}_r) dB_t$$

For the weak dependence of the noise velocity $\vec{v}_N$ on the external electric field $\vec{\mathrm{E}}_{Ext}(V_{bias}, f_{MIX}(S_D(t), S_{IN}(t))$ and the mean fluctuation force, and the relatively strong dependence of the noise velocity to the internal electric field $\vec{\mathrm{E}}_{Int}(\vec{x},t)$, the coefficients α_{N1}, α_{N2} and α_{N3} are used in the following approximation,

$$d\vec{v}_N = \frac{-q\alpha_{N1}\vec{\mathrm{E}}_{Int}(\vec{x},t) - q\alpha_{N2}\vec{\mathrm{E}}_{Ext} - \alpha_{N3}\lambda(0) m_e \vec{v}_N}{m_e} dt + \frac{\sigma(\vec{F}_r)}{m_e} dB_t \tag{I.11}$$

$$d\vec{v}_N = \mu(\vec{v}_N) dt + \frac{\sigma(\vec{F}_r)}{m_e} dB_t$$

The instantaneous noise power $\sigma_N^2(t)$ in the transport path direction with noise velocity v_N is approximated by the following expression,

$\sigma_N^2(t) \approx C_N q^2 n^2 v_N^2$, where C_N is a constant, q is electronic charge and n is the electronic density

By using stochastic calculus,

$$d\sigma_N^2 \approx \left[\frac{d\sigma_N^2}{dv_N}\mu(v_N) + 0.5\frac{d^2\sigma_N^2}{dv_N{}^2}\frac{\sigma^2(\vec{F}_r)}{m_e^2}\right] dt + \left[\frac{d\sigma_N^2}{dv_N}.\frac{\sigma(\vec{F}_r)}{m_e}\right] dB_t \tag{I.12}$$

$$\approx \left[2C_N q^2 n^2 v_N \mu(v_N) + C_N q^2 n^2 \frac{\sigma^2(\vec{F}_r)}{m_e^2}\right] dt + \left[2C_N q^2 n^2 v_N . \frac{\sigma(\vec{F}_r)}{m_e}\right] dB_t = \mu(\sigma_N^2) dt + \varsigma(\sigma_N^2) dB_t$$

The pseudo randomness of the instantaneous noise power $\sigma_N^2(t)$ is evident from the drift contribution $\mu(\sigma_N^2)$ and the Markovian state machine can be constructed by limited states due to semi-ballistic transport and relatively lower variance of the random scattering $\sigma^2(\vec{F}_r)$.

The Markovian state machine parameters, i.e. observable symbol probability distributions, $b_j(k)$ and the state transition distributions $a_{ij}(k)$ are related to the time evolutionary Fokker Planck probability distribution $P(\sigma_N^2, t)$,

$$\frac{\partial P(\sigma_N^2, t)}{\partial t} = -\frac{\partial}{\partial \sigma_N^2}[\mu(\sigma_N^2)P(\sigma_N^2, t)] + 0.5\frac{\partial^2}{\partial {\sigma_N^2}^2}[\varsigma(\sigma_N^2)P(\sigma_N^2, t)] \tag{I.13}$$

The Milstein method [34] for estimating $\sigma_N^2(n)$ from $\sigma_N^2(n-1)$ is expressed below:

$$\sigma_N^2(n) = \sigma_N^2(n-1) + \mu(\sigma_N^2, n)\Delta T + \varsigma(\sigma_N^2, n).\Delta B_t(n-1) + 0.5\varsigma(\sigma_N^2, n)\varsigma'(\sigma_N^2, n)\left(\Delta B_t^2(n-1) - \Delta T\right) \tag{I.14}$$

where the derivative is performed with respect to time.

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